Characterization of magnitude distributions in induced seismicity settings

How much can the data tell us?

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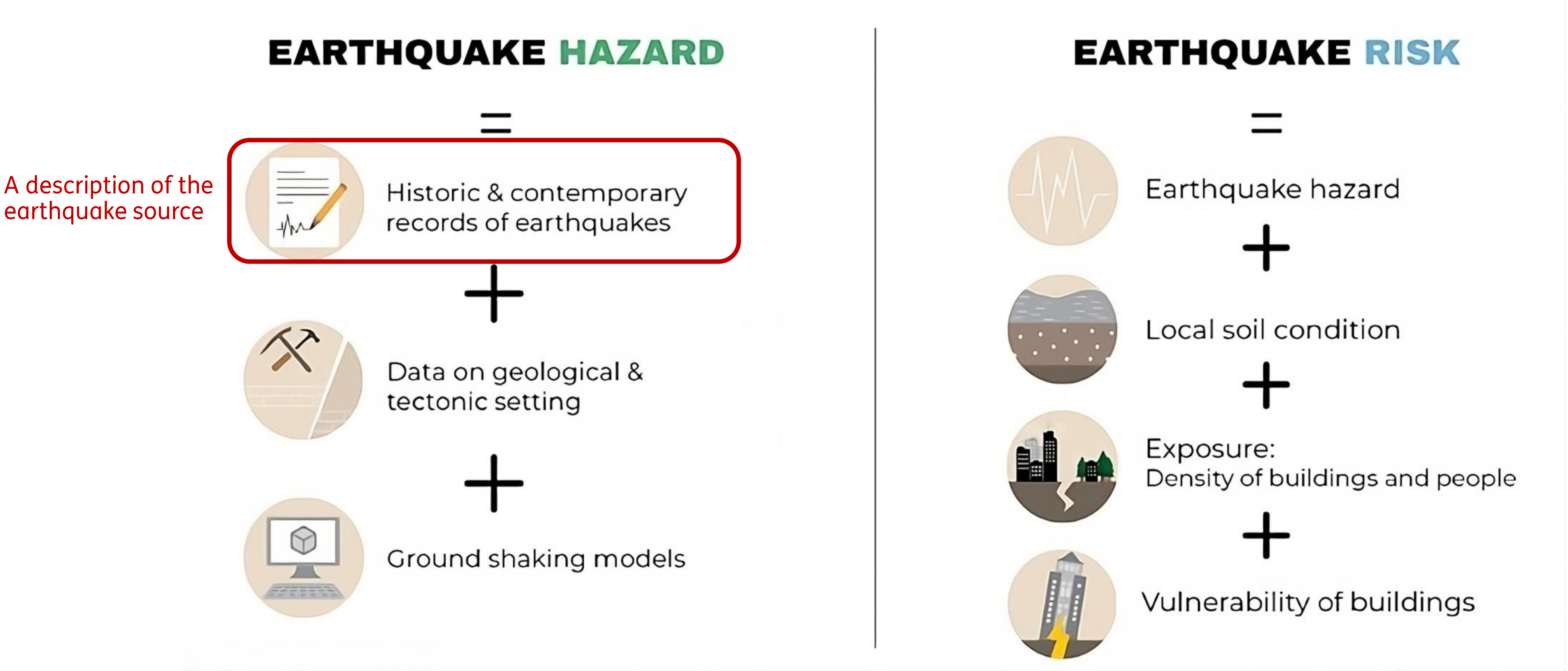
What to expect from this talk:

1. **A plea to stop using best-fit models (point estimates) for forecasting seismicity in general and frequency-magnitude distributions in particular**

2. **A demonstration:**
   - **Why this is so important when estimating a corner magnitude for a tapered magnitude distribution**
   - **Why you (kind of) get away with point estimates for b-value estimation**

- **But first**
  - Why do we care in the first place?
  - What is a tapered magnitude distribution?

Why do we care about describing earthquake sources?



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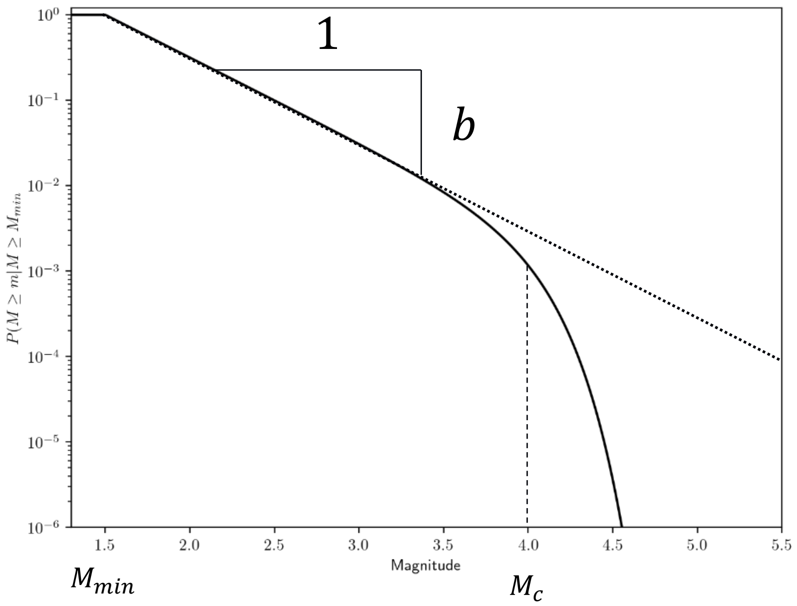
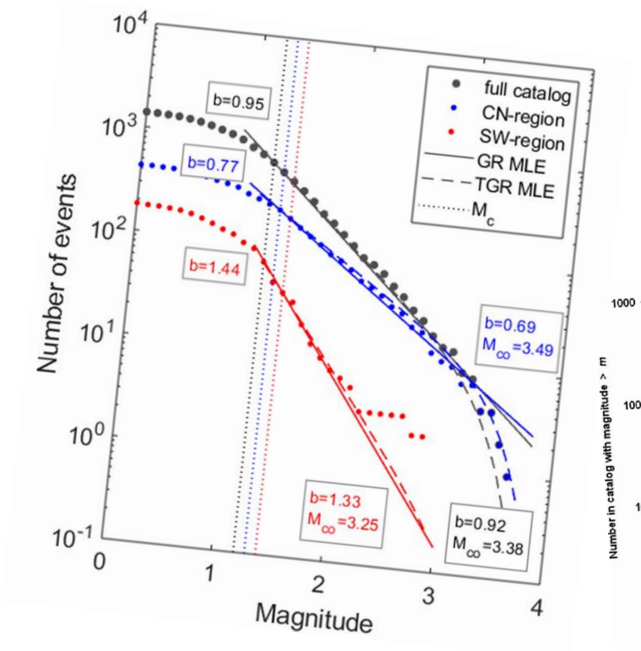
What is a tapered magnitude distribution?

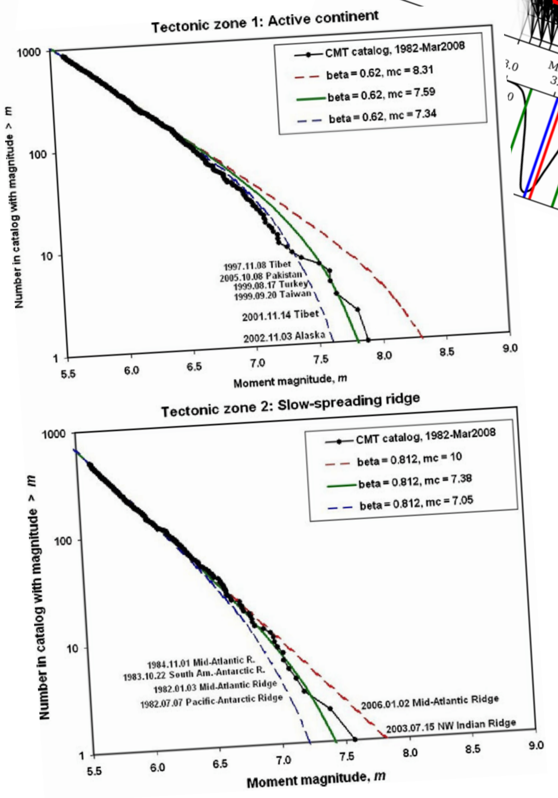
1. Earthquake magnitudes for a given system *tend to be* distributed exponentially (Gutenberg-Richter)
2. Because faults have a finite size, and energy budgets are finite, the G-R distribution *cannot* be valid for the entire magnitude range up to ∞.
3. A physically plausible and popular choice to describe the deviation from a classical G-R distribution: a tapered Gutenberg-Richter distribution
4. Earthquake hazard and risk are largely controlled by the large earthquakes (the rare earthquakes, the earthquakes in the tail of the magnitude distribution). **So it's important to get the description of the tail right!**

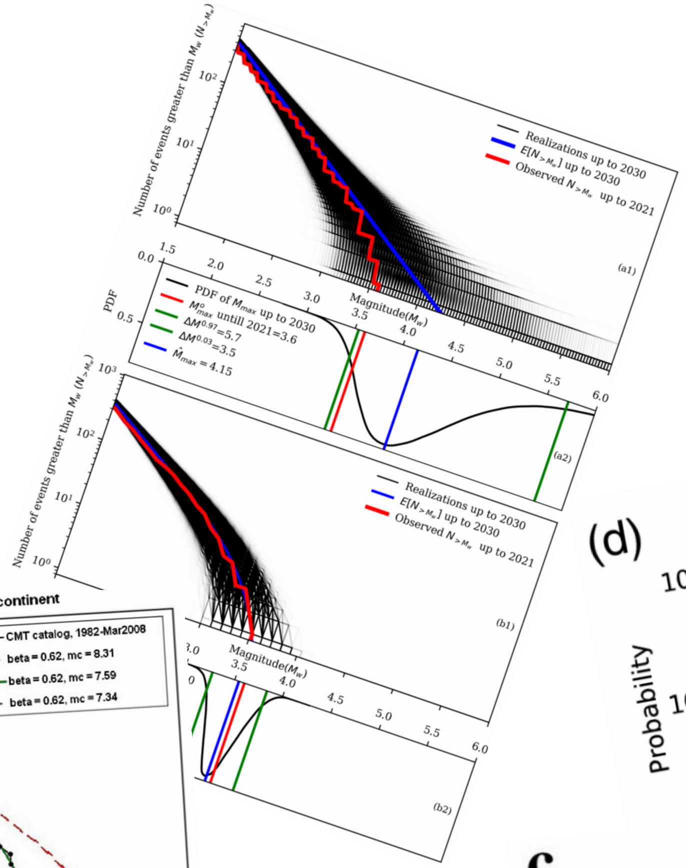


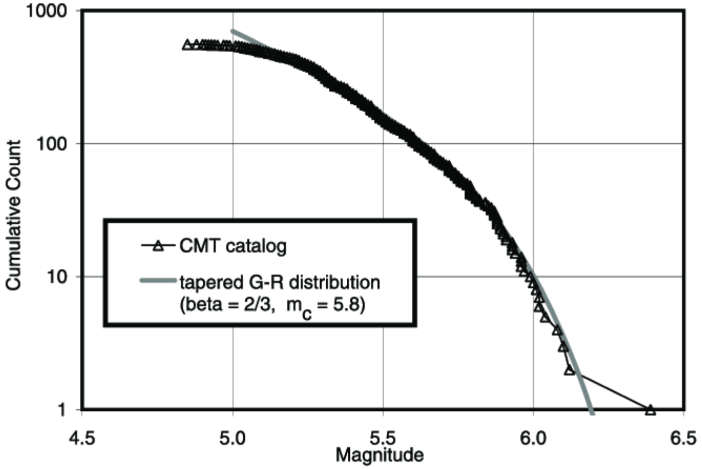
$$P(M \geq m \mid M \geq M_{min}) = 10^{-b(m-M_{min})} \times e^{-10^{\frac{3}{2}(m-M_C)}}$$

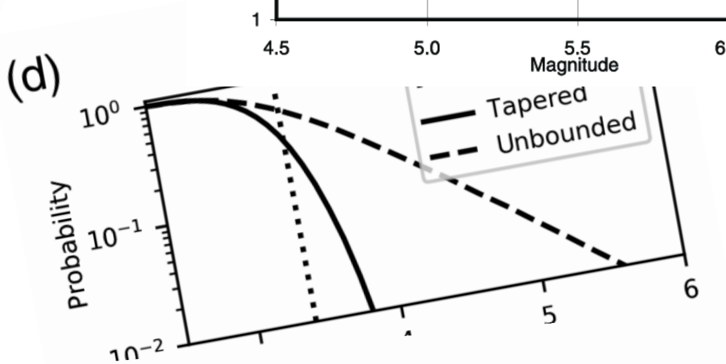
Some examples

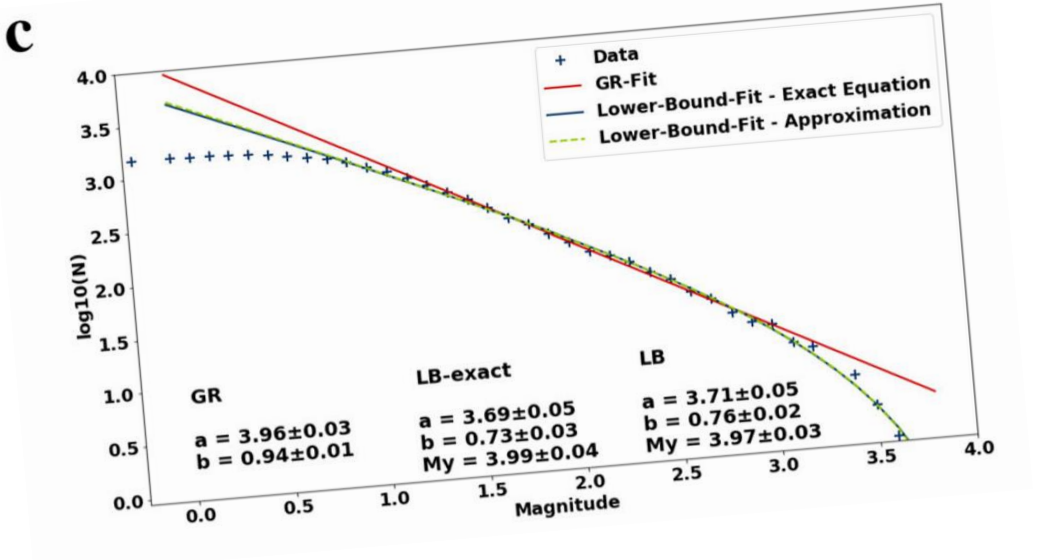












A demonstration

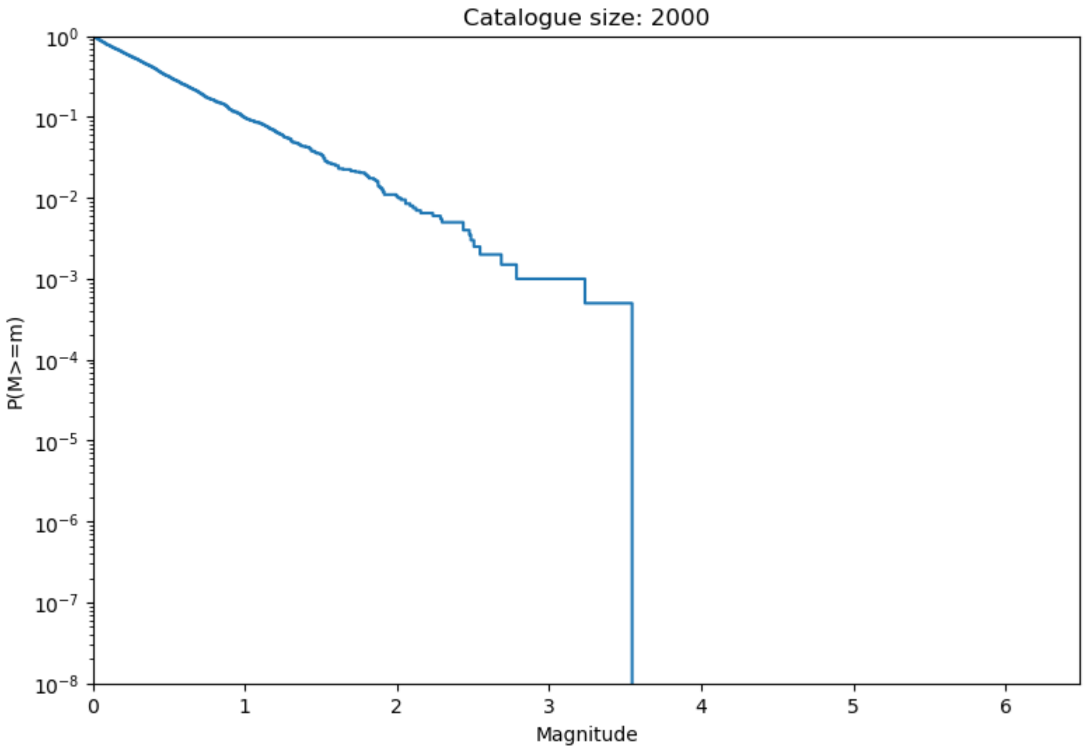
- **Key question: How good are we at estimating the *true magnitude distribution* from an observed catalogue?**
  - In real cases, we don't know the 'true distribution' (that's why we estimate it)
  - We don't even know if a (tapered) Gutenberg-Richter is an appropriate model to describe the earthquake source
  - But we do typically use these models in our forecasts
  - So **at the very least**, we should know how good we are at estimating the true magnitude distribution when we **know** that the EQ source *is in fact* a (tapered) Gutenberg-Richter distribution.

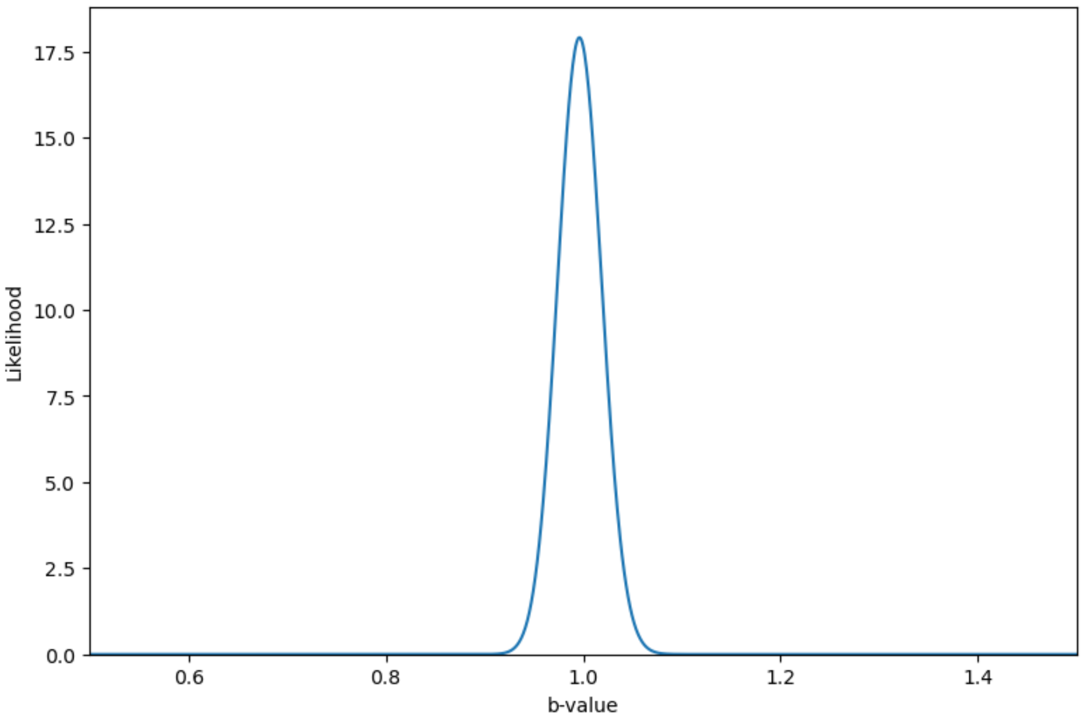
- Let's investigate our success inferring the true magnitude distribution from synthetic catalogues with a known ground truth. That way we can actually check our answers
  - Note that we're **not** interested in retrieving the true values of b or M_c but rather in retrieving the **true magnitude distribution** (e.g. how good are we at estimating the probability of exceedance of M5.0?)

A demonstration

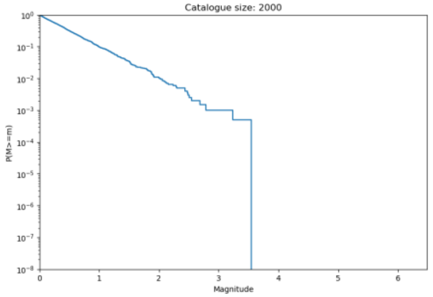
- ✓ Complete
- ✓ Magnitudes are infinitely accurate
- ✓ 2000 events

How do I base a forecast on this likelihood distribution of b?





A demonstration



$$\underbrace{P(b|\boldsymbol{M})}_{\text{Posterior probability}} \sim \underbrace{P(\boldsymbol{M}|b)}_{\text{Likelihood}} \times \underbrace{P(b)}_{\text{Prior probability}}$$

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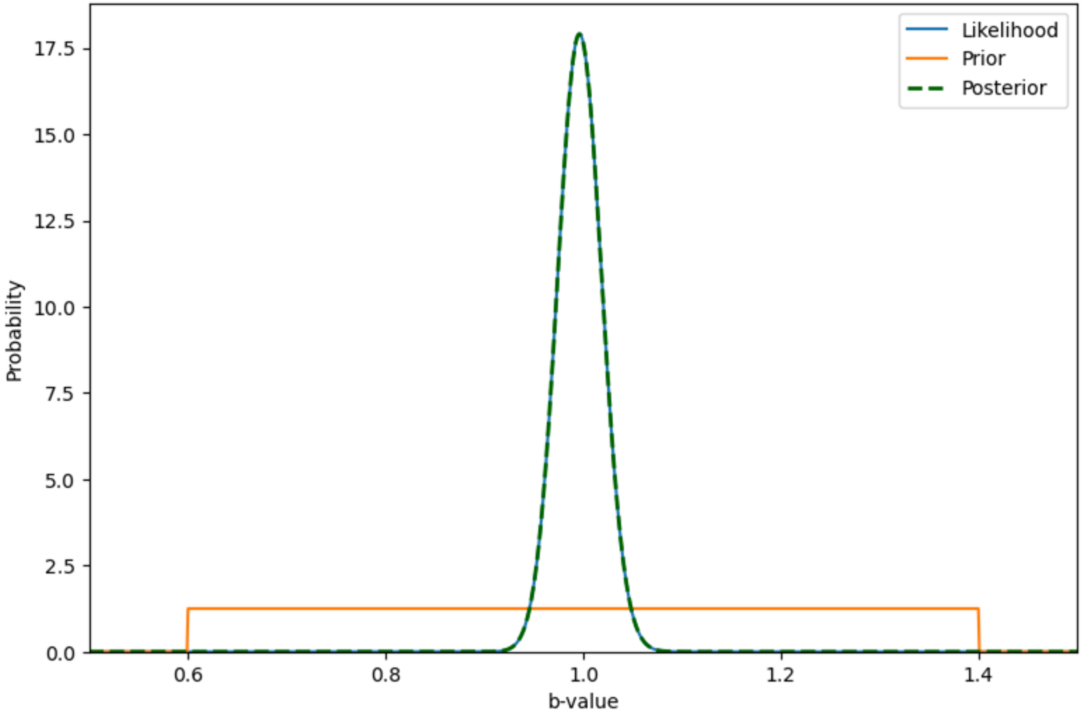

A demonstration

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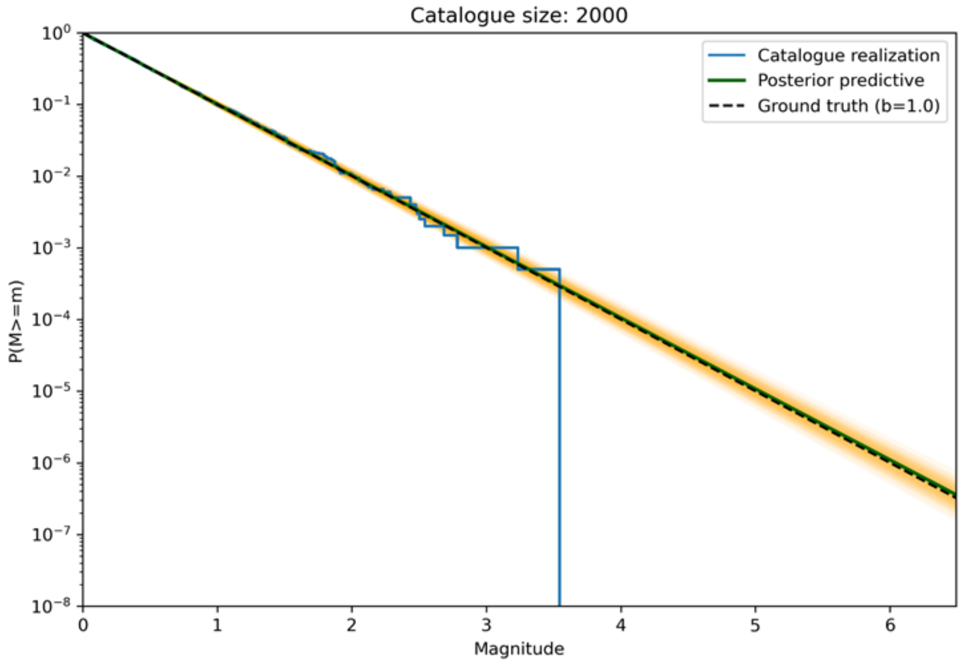
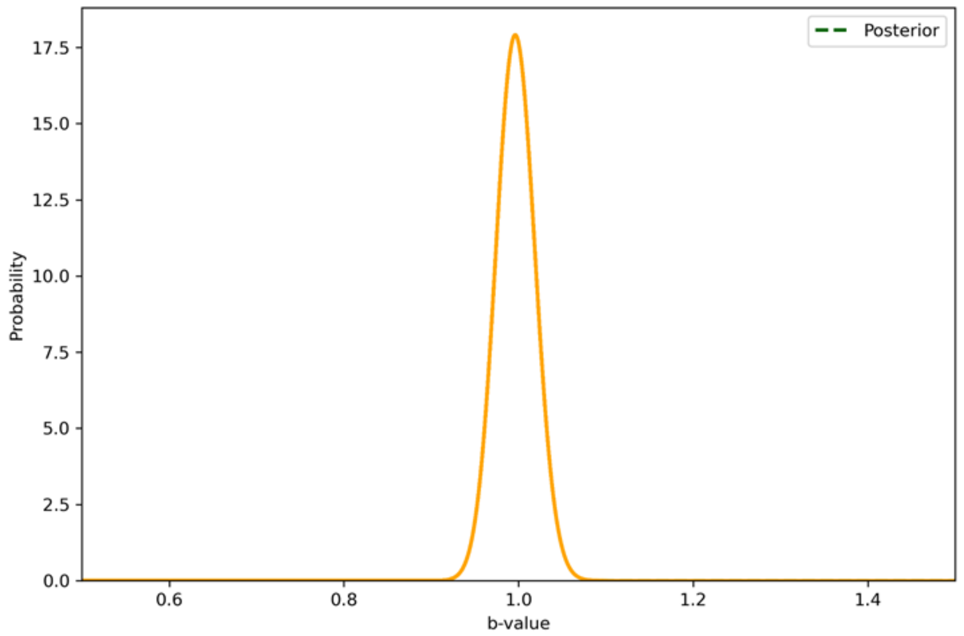
- State of the art (for decades) is to assess seismic hazard and risk *probabilistically:* We want our forecast to take into account all **plausible** models **consistent** with the observations, **weighted** by their respective probabilities: **the Posterior Predictive Magnitude Distribution**





A demonstration

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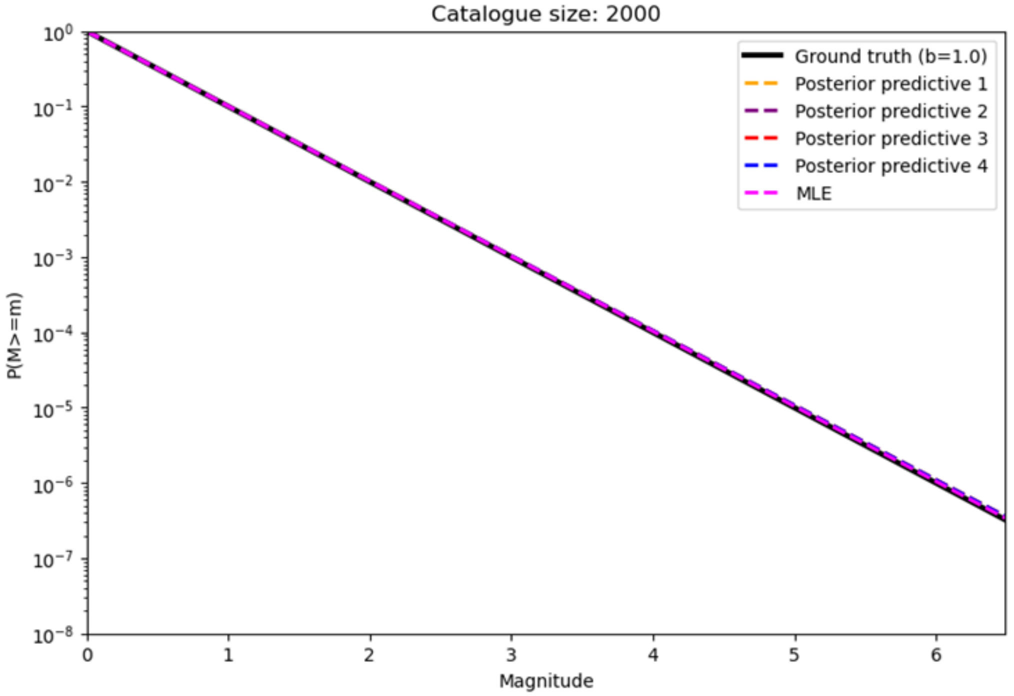
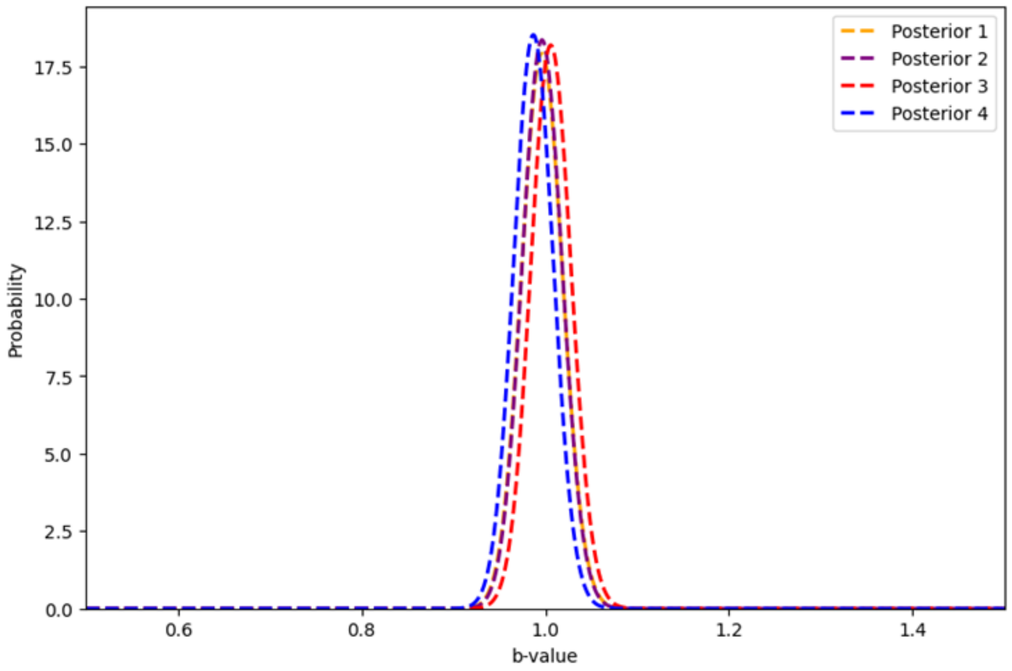




A demonstration

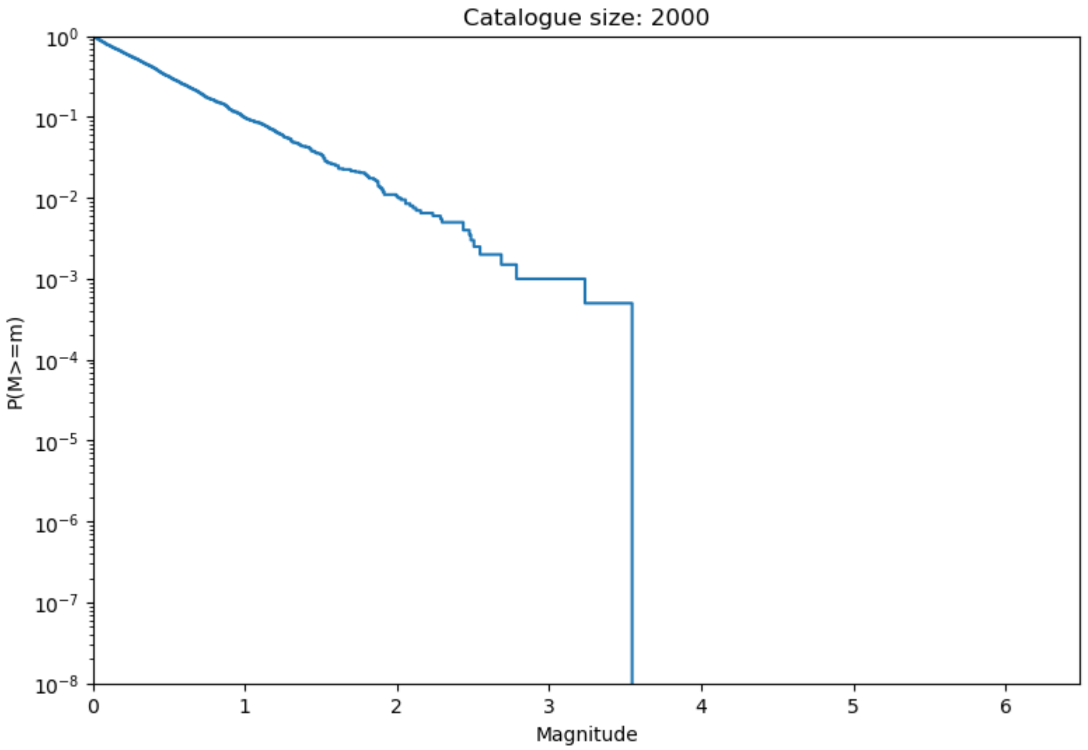
	Slope parameter b
Importance of prior	Minimal
Importance of data	Large
Performance of PPMD	Very good
Performance of MLE	Surprisingly good*
Shape of likelihood distribution	Symmetrical

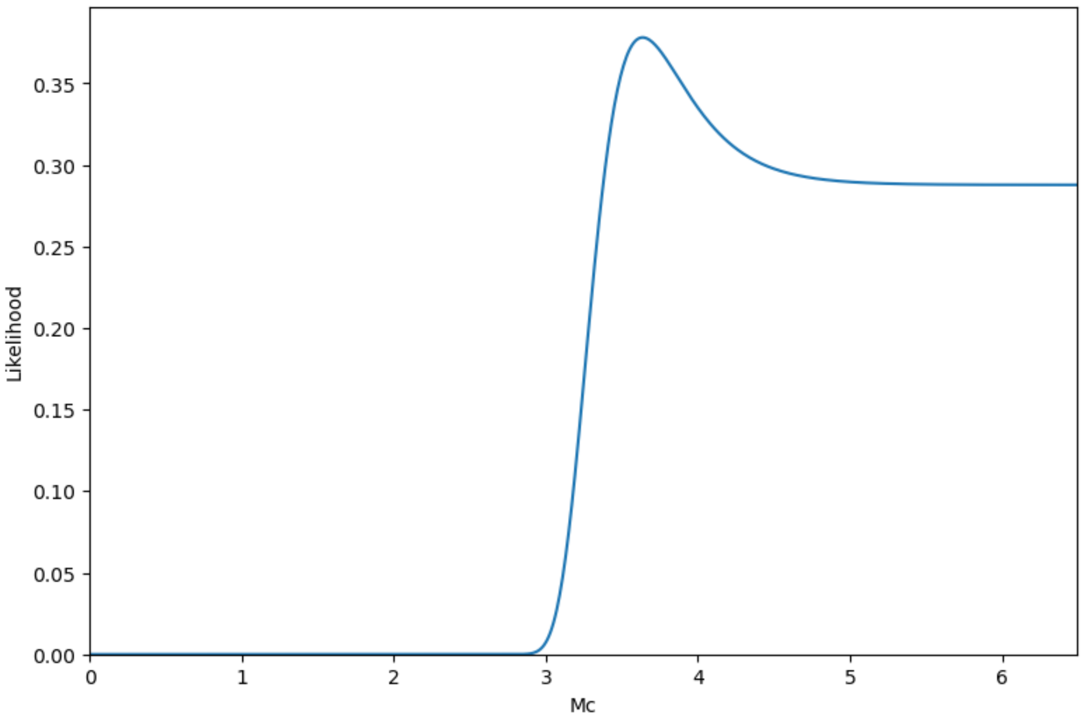
* For a reasonable size catalogue. (We still shouldn't use it)

A demonstration

- ✓ Complete
- ✓ Magnitudes are infinitely accurate
- ✓ 2000 events

How do I base a forecast on this likelihood distribution of M_c?

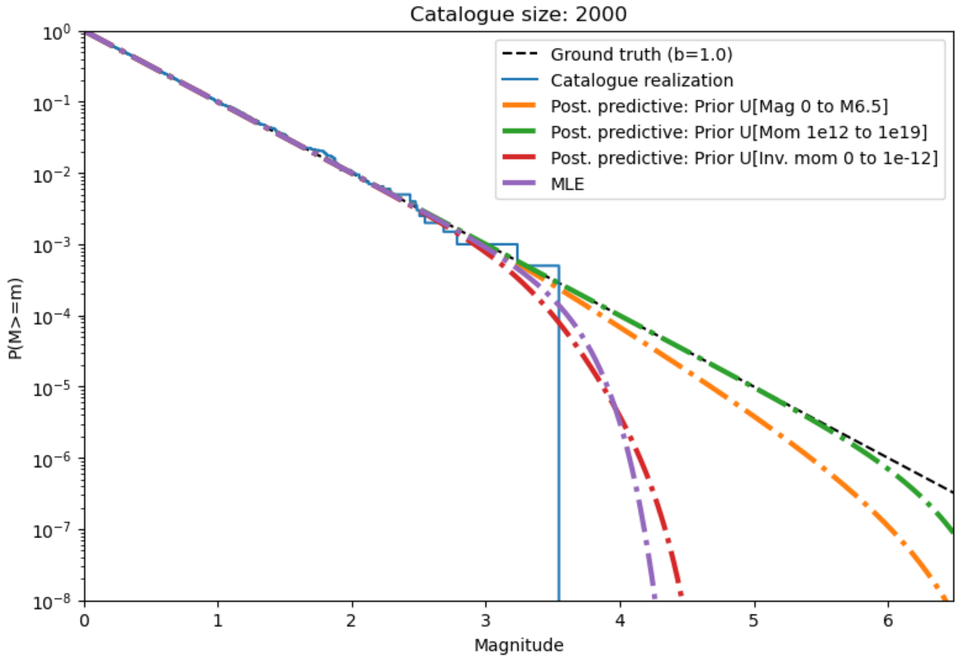




A demonstration

- All we have to do is choose a prior distribution!
- A uniform distribution?
  - $U[\text{M}0.0\ ;\ \text{M}6.5]$: Uniform in moment magnitude
  - $U[\mathcal{M}10^{12}\ ;\ \mathcal{M}10^{19}]$: Uniform in seismic moment
  - $U[\zeta 0\ ;\ \zeta 10^{-12}]$: Uniform in inverse seismic moment

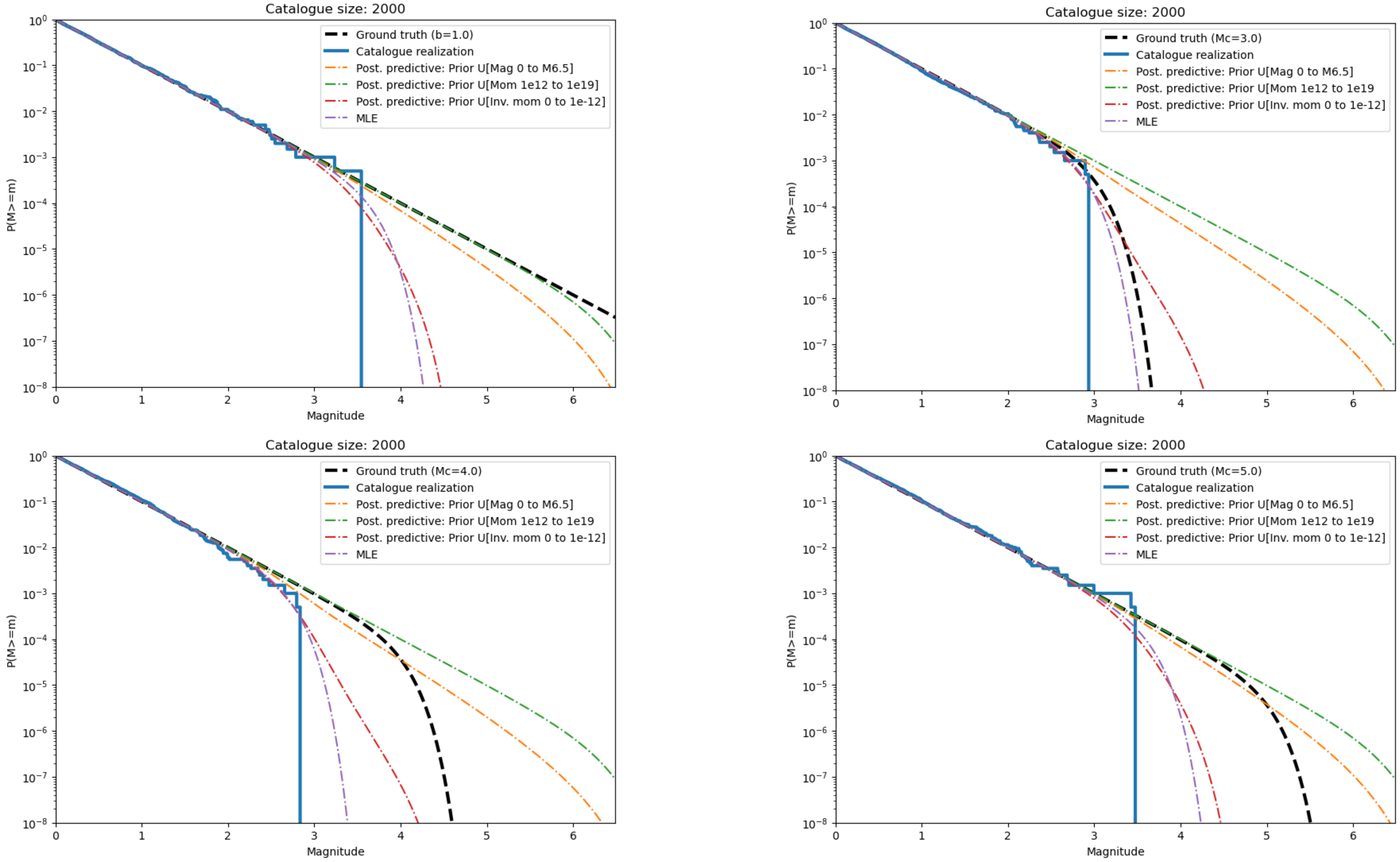
$$\underbrace{P(b|\boldsymbol{M})}_{\text{Posterior probability}} \sim \underbrace{P(\boldsymbol{M}|b)}_{\text{Likelihood}} \times \underbrace{P(b)}_{\text{Prior probability}}$$



MLE
M4+: factor 35 underestimation
M4.5+ factor 56 million underestimation

Different ground truths

- MLE systematically underestimates the true distribution
- Posterior predictive models don't change for different ground truths: they are dominated by the prior, because the data doesn't contain much information about the tail



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A demonstration

	Slope parameter b	Taper parameter M_c
Importance of prior	Minimal	Huge
Importance of data	Large	Only rules out low values of M_c
Performance of PPMD	Very good	Only good if your prior was good
Performance of MLE	Surprisingly good	Almost universally bad
Shape of likelihood distribution	Symmetrical	Asymmetrical

In summary, I've presented

1. A plea to **stop using best-fit models** (point estimates) for forecasting seismicity in general and frequency-magnitude distributions in particular

2. A demonstration:
   - Why this is so important when estimating a corner magnitude for a tapered magnitude distribution
   - Why you (kind of) get away with point estimates for b-value estimation

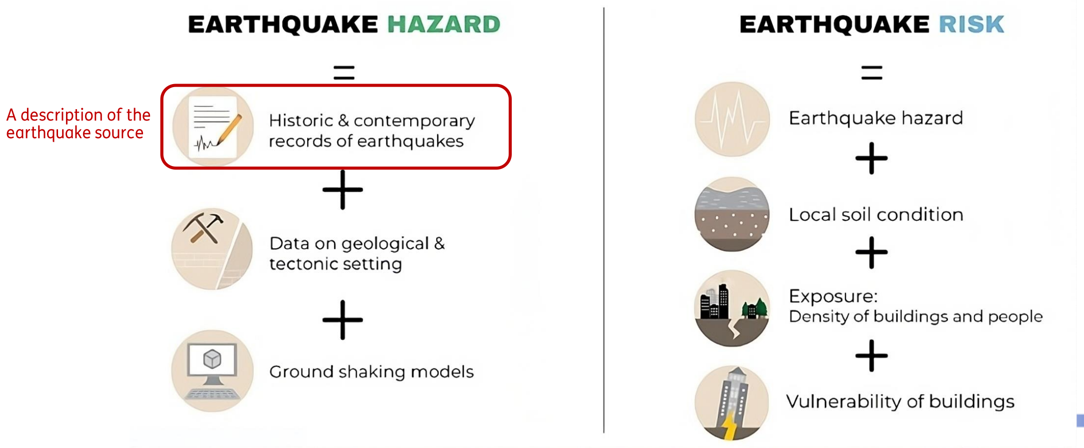
3. What to instead:
   - Use a **Posterior Predictive model** which takes into account all **plausible** models **consistent** with the observations, **weighted** by their respective probabilities
   - This requires **thinking about and justifying** a prior distribution, which can be difficult to do. **This is where we need scientific progress and discussion!**

The extremely condensed take-away

1. 'Best-fit model' is a generous synonym for **'least terrible single model'**. But using a single model to forecast is a terrible idea in itself.

2. If the choice of prior matters... the choice of prior **matters!**

3. Even if a model isn't directly feeding into an SHRA, these aspects shouldn't be ignored. Our work has societal relevance and impact!



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Thank you

Questions?