On the equivalent Biot and Skempton coefficients of fractured rocks and their impact on the HM behavior of geological media

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Hossein A. Kasani

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Biot and Skempton coefficients govern HM rock behavior

- Effective stress $\Delta\sigma'$ controls deformation of saturated porous materials because part of the total load $\Delta\sigma$ acts on the fluid pressure Δp
- In **undrained conditions** (or in the short-time), an increase in load (total stress, $\Delta\sigma$) causes a proportional increase in fluid pressure Δp
- The product $\boldsymbol{\alpha}\,\boldsymbol{B}$ defines the impact of an applied load on the solid skeleton, and thus the material deformation, under undrained conditions.

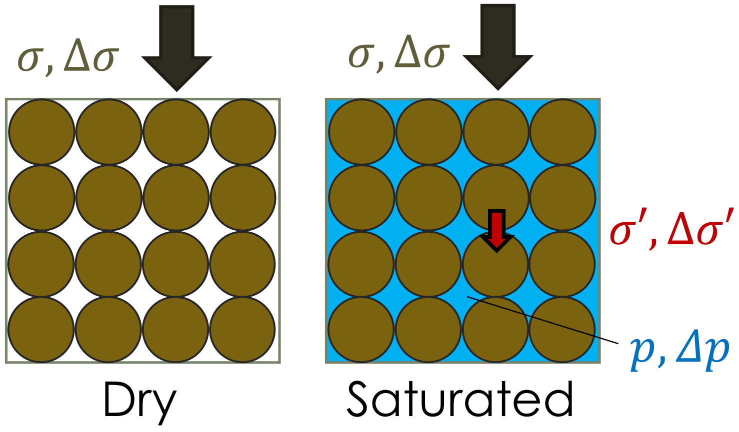
$$\sigma' = \sigma - \alpha\, p$$
$$\Delta\sigma' = \Delta\sigma - \alpha\, \Delta p$$

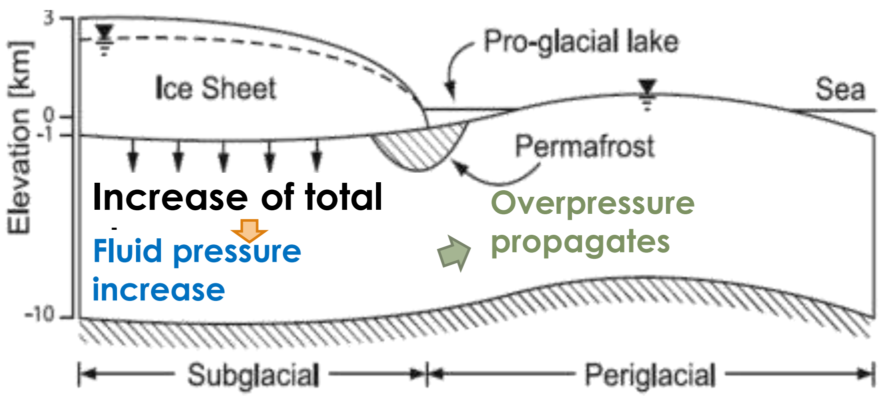
Biot coefficient $\boldsymbol{\alpha}$

$$\Delta p = \boldsymbol{B}\Delta\sigma$$

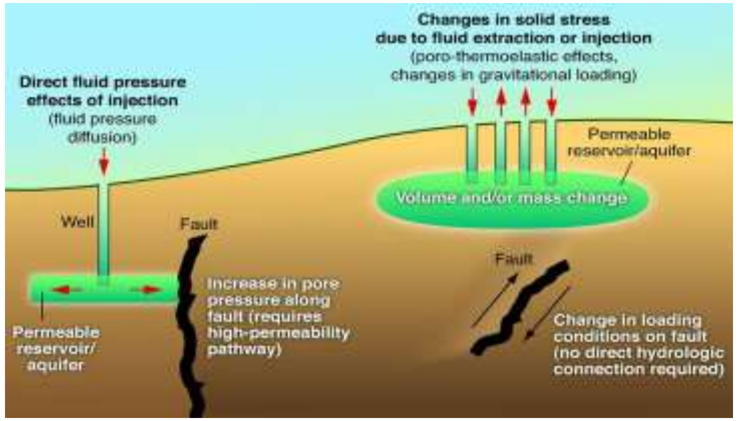
Skempton coefficient $\boldsymbol{B}$

$$\Delta\sigma' = (1 - \boldsymbol{\alpha B})\Delta\sigma$$





Modified from Lemieux et al. (*JGR*, 2008)



Ellsworth (*Science*, 2013)

Some details about the poroleastic coefficients (*)

- Biot coefficient $\boldsymbol{\alpha}$ reflects the effects of pore fluid pressure on the solid matrix
- It depends on the petrophysical properties of the solid skeleton at the microscale

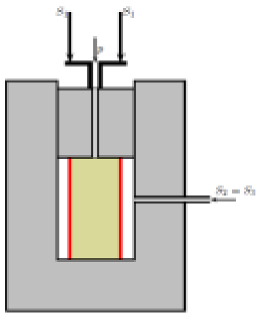
$$\sigma' = \sigma - \boldsymbol{\alpha}\, p$$

$$\alpha = \left(\frac{\partial \sigma_m}{\partial p}\right)_V$$

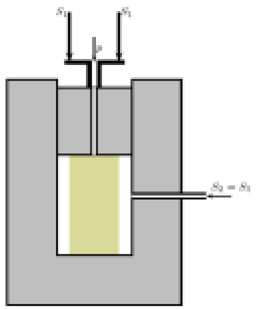
- Skempton coefficient $\boldsymbol{B}$ defines the pressure variation in response to total stress variation under undrained conditions
- It depends on rock and fluid properties at bulk scale

$$B = \left(\frac{\partial p}{\partial \sigma_m}\right)_\zeta$$

- They range between 0 and 1, being ≈ 1 in highly compressible materials (e.g., soils), and < 1 in stiff formations (e.g., crystalline rocks);
- They can be directly or indirectly measured in the lab, or they can be calculated from known properties through theoretical expressions valid for homogeneous isotropic media



Jacketed test

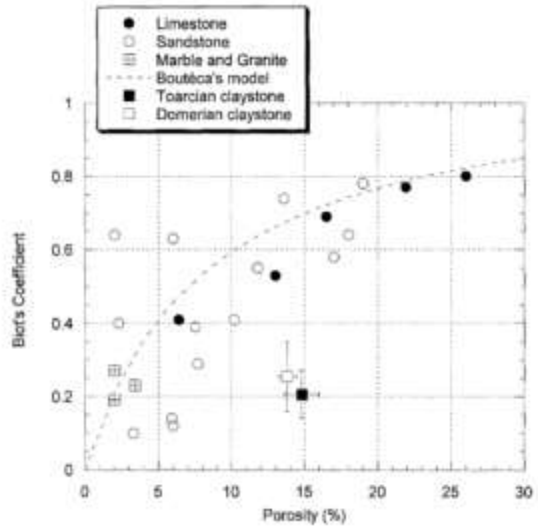


Unjacketed test

* Refs: Biot 1941; see also: Detournay & Cheng 1993; Cheng 2016; Coussy 2004

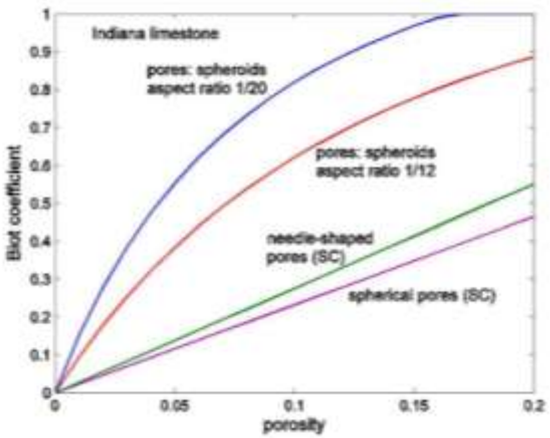
The coefficients largely depend on pores and their shape

α increases with **porosity**



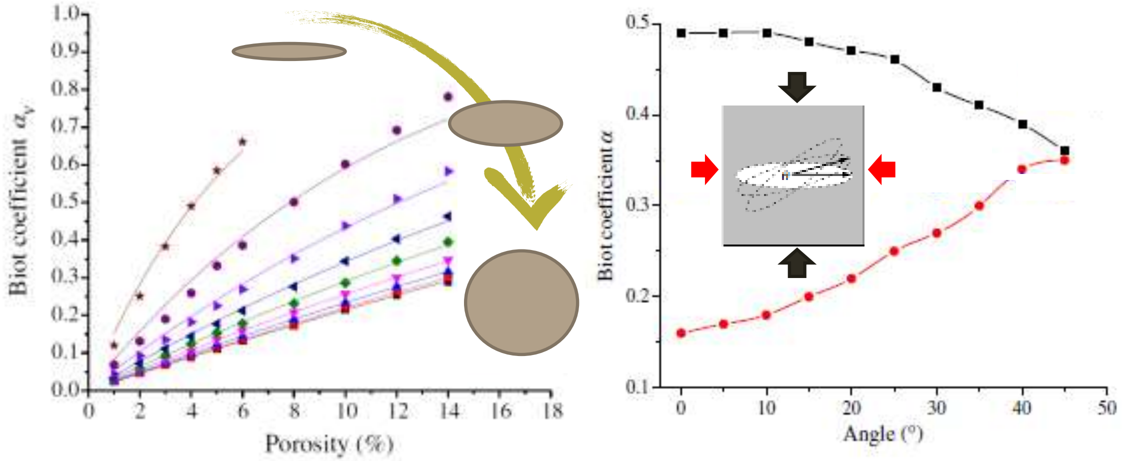
Experimental data
(Selvadurai, *Geosciences*, 2021)

α increases with **elongated** pores and cracks



Theoretical estimations
(Selvadurai & Suvorov, *Sci. Rep.*, 2020)

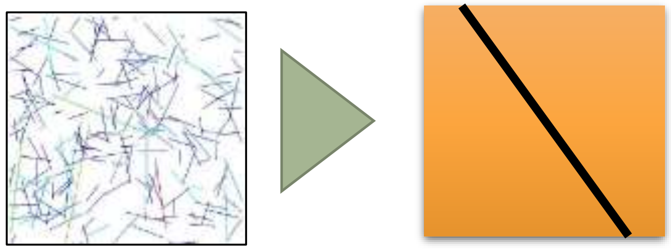
α is **anisotropic**



Numerical estimations
(Modified from Tan & Konietzky, *Tecton.*, 2014)

† studies limited to sample scale

What about large-scale fractured rocks?



Field-scale problems require estimating effective poroelastic coefficients referring to the rock mass

Proposed estimations for fractured rocks:

- Poroelasticity theory for anisotropic porous media (e.g., Cheng1997+ extensive literature)
- Equivalent stiffness for fractured rocks + traditional theoretical expressions (e.g., Wong 2017; Selvadurai et al. 2019; Selvadurai and Suvorov 2020; Berryman 2012)
- Numerical estimations from observed response (e.g., Chen et al. 2020)
- Volume-weighted averaging (e.g., Tuncay&Corapcioglu 1995; Tan&Konietsky 2014)

☹ Limited to sample scale

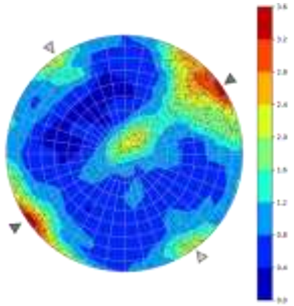
☹ Limited to 2D

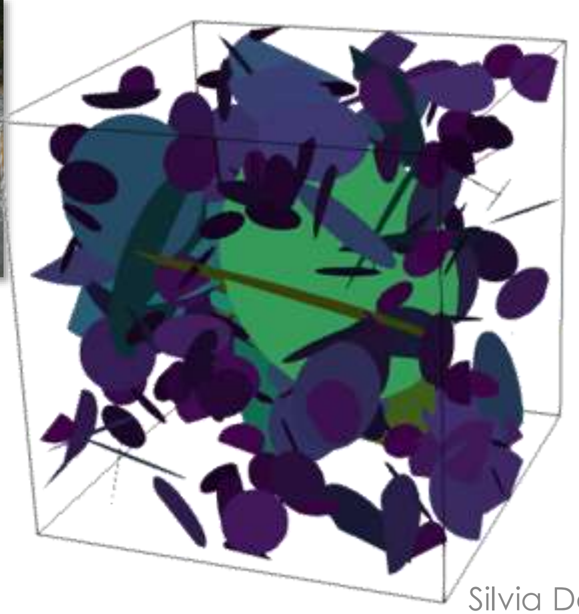
☹ Not validated

☹ No consideration of fracture orientation

☹ No consideration of fracture volume

☹ No porosity of intact rock

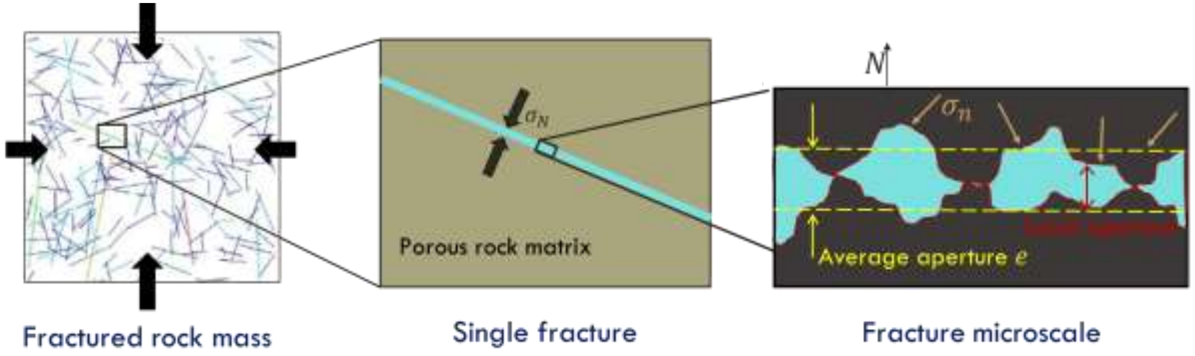


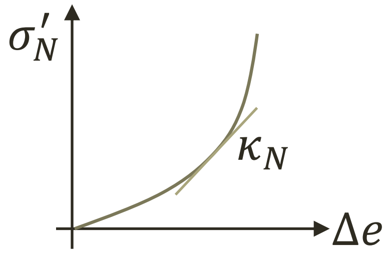


Defining equivalent Biot and Skempton coefficients for a single saturated fracture

Assumptions:

- Fracture with aperture e is **fluid-filled**
- Fracture characteristics (aperture, stiffness) are **homogeneous in the plane**
- The resulting force is perpendicular to the average fracture plane (deviations due to asperities cancel out)
- Fracture behavior is locally linear elastic with **normal stiffness κ_N**





σ_N = normal stress

$$\Delta\sigma'_N = \Delta\sigma_N - \alpha^f \Delta p$$

$$\Delta p = B^f \Delta\sigma_N$$

Fracture Biot coefficient

- $\alpha^f = 1$ for open fractures, while $\alpha^f = 0$ for sealed fractures.

Fracture Skempton coefficient

- analytically derived considering a volumetric approach $B^f = (\beta e \kappa_N + \alpha_N)^{-1}$,
  β= fluid compressibility

Conceptual model to define equivalent Biot and Skempton coefficients for a saturated fractured rock mass

$\Delta\sigma'_m = \Delta\sigma_m - \bar{\alpha}\Delta p$, **equivalent Biot coeff.**

$\Delta p = \bar{B}\Delta\sigma_m$, **equivalent Skempton coeff.**

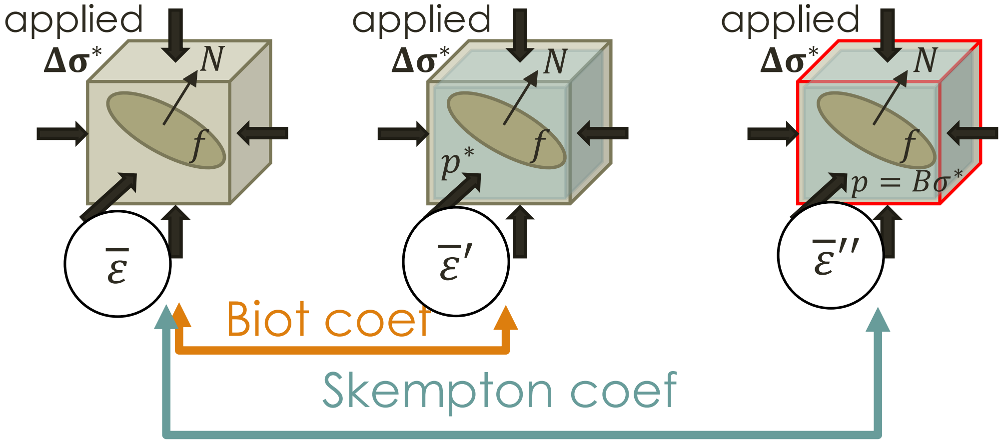
$\Delta\sigma_m$= average stress

- Equivalent coefficients are those that control the effective deformation.
- They are derived by comparing the **total volume deformations** under an applied incremental stress tensor $\boldsymbol{\Delta\sigma}^*$ and different hydraulic conditions.

Dry $\boldsymbol{p = 0}$

Drained imposed $\boldsymbol{\Delta p^*}$

Undrained $\boldsymbol{\Delta p = B\,\Delta\sigma^*_m}$



$$\frac{\bar{\varepsilon}}{\Delta\sigma^*_m} = \frac{\bar{\varepsilon}'}{\Delta\sigma^*_m - \bar{\alpha}\,\Delta p^*} \;\Rightarrow\; \bar{\alpha} = \frac{\bar{\varepsilon} - \bar{\varepsilon}'}{\bar{\varepsilon}}\frac{\Delta\sigma^*_m}{\Delta p^*}$$

$$\frac{\bar{\varepsilon}}{\Delta\sigma^*_m} = \frac{\bar{\varepsilon}''}{(1-\bar{\alpha}\,\bar{B})\Delta\sigma^*_m} \;\Rightarrow\; \bar{B} = \frac{\bar{\varepsilon} - \bar{\varepsilon}''}{\bar{\varepsilon}}\frac{1}{\bar{\alpha}}$$

Defining equivalent Biot and Skempton coefficients for a saturated fractured rock mass

$\Delta\sigma'_m = \Delta\sigma_m - \overline{\alpha}\Delta p$, **Biot coeff.**

$\Delta p = \overline{B}\Delta\sigma_m$, **Skempton coeff.**

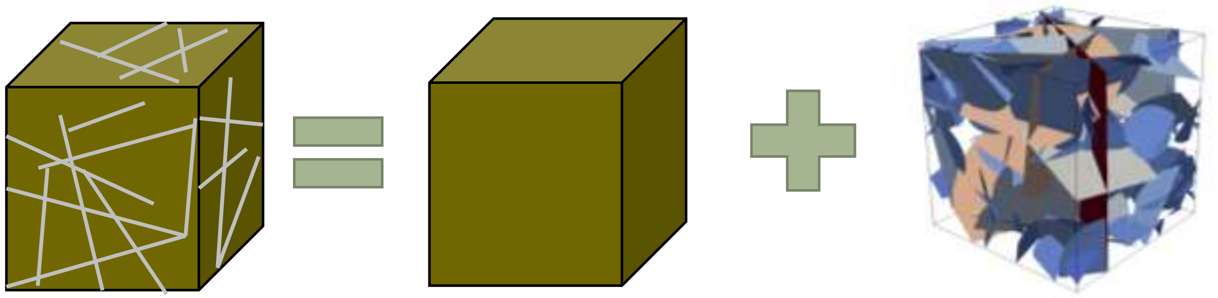
$\Delta\sigma_m$= average stress

$$\overline{\alpha} = \frac{\overline{\varepsilon} - \overline{\varepsilon}'}{\overline{\varepsilon}} \frac{\Delta\sigma_m^*}{\Delta p^*}$$

$$\overline{B} = \frac{\overline{\varepsilon} - \overline{\varepsilon}''}{\overline{\varepsilon}} \frac{1}{\overline{\alpha}}$$

Total volume variation $\Delta V = \Delta V^r + \sum_f \Delta V^f$

(non-interaction approximation - Grechka & Kachanov, 2006))



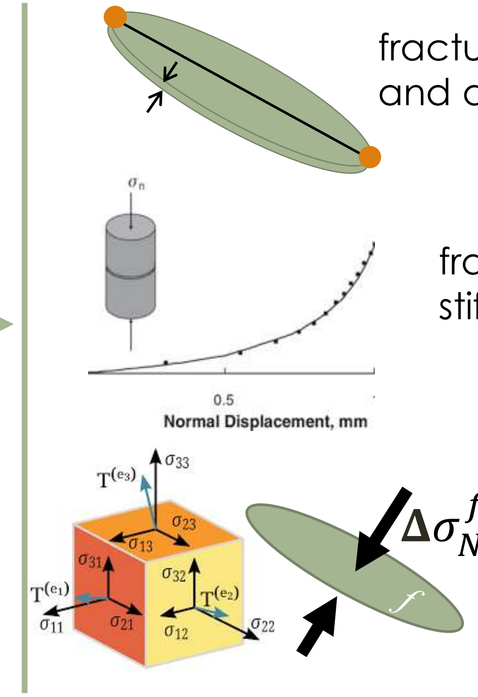
Intact rock (porous)

Fracture network

γ^r, α^r, B^r

$\gamma^i = \Delta V^i / |\boldsymbol{\Delta\sigma}^*|$

contribution of element i to the total deformation



fracture size and aperture

γ^f, α^f, B^f

fracture normal stiffness

Initial and applied stress on the fracture

$$\theta^f = \frac{\Delta\sigma_N^f}{\Delta\sigma_m} = \frac{n_f^T \boldsymbol{\Delta\sigma}\, n_f}{tr(\boldsymbol{\Delta\sigma})/3}$$

$$\overline{\alpha} = \frac{\gamma^r \alpha^r + \sum_f \gamma^f \alpha^f}{\gamma^r + \sum_f \theta^f \gamma^f}$$

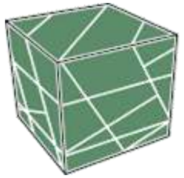
$$\overline{B} = \frac{\gamma^r \alpha^r B^r + \sum_f \gamma^f \theta^f \alpha^f B^f}{\gamma^r \alpha^r + \sum_f \gamma^f \alpha^f}$$

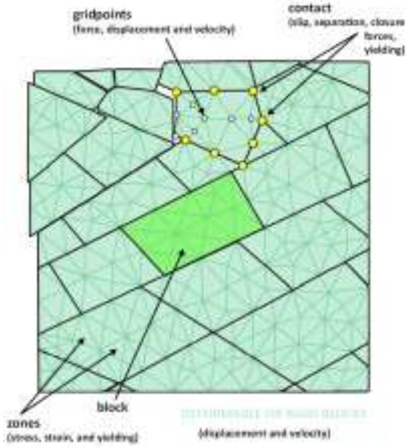
- **γ-weighted averages**
- **Depend on the load $\boldsymbol{\Delta\sigma}$**

(De Simone *et al.*, RMRE 2023)

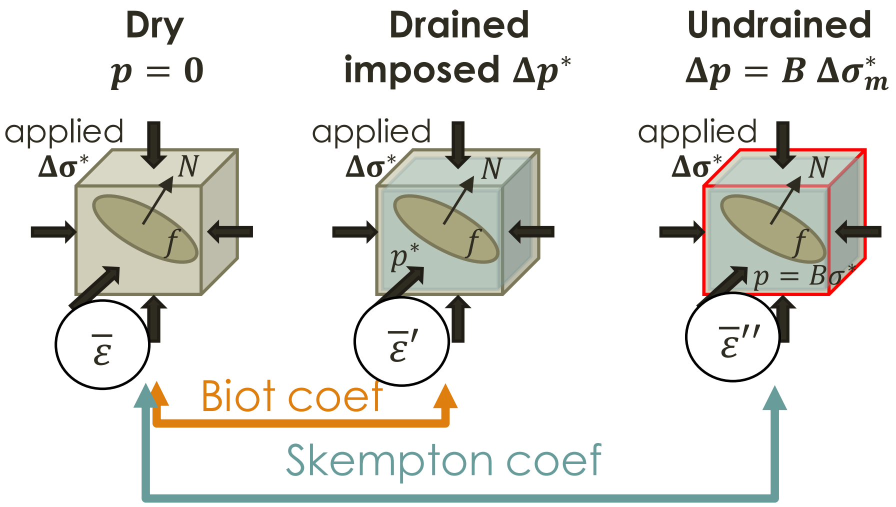
Validated against numerical results (3DEC)

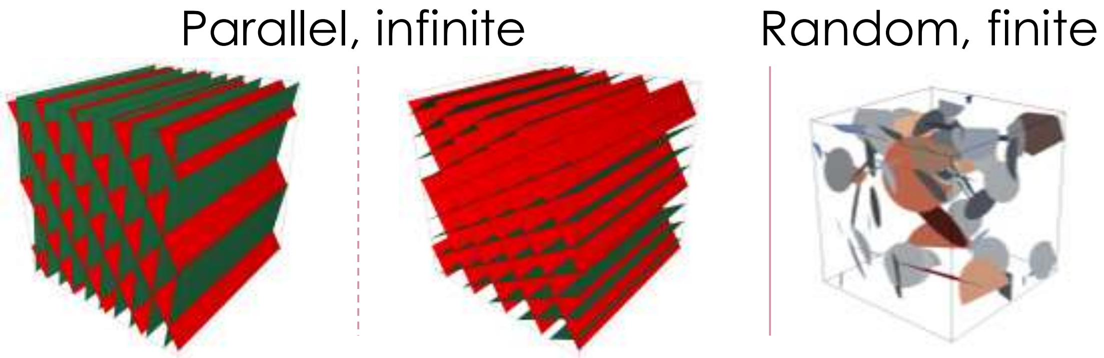
- Fractures and rock blocks are explicitly represented, and their HM behavior simulated
  - Distinct Element Method
  - Assembly of deformable blocks limited by fracture planes
  - Elastic behavior inside fracture plane
  - Loaded by stress





- The three hydraulic conditions are reproduced, and the total volume variations estimated
- Different fracture settings
- Stress is alternatively applied in the 3 directions





(De Simone *et al.*, RMRE 2023)

Variability of equivalent coefficients when fracture aperture (e) and stiffness (κ_N) are constant (i)

$$\overline{\alpha} = \frac{\gamma^r \alpha^r + \sum_f \gamma^f \alpha^f}{\gamma^r + \sum_f \theta^f \gamma^f}$$

$$\sum_f \gamma^f = \sum_f \frac{S^f}{\kappa_N^f} = p_{32}\, \kappa_{n,f}^{-1}$$

$$\overline{B} = \frac{\gamma^r \alpha^r B^r + \sum_f \gamma^f \theta^f \alpha^f B^f}{\gamma^r \alpha^r + \sum_f \gamma^f \alpha^f}$$

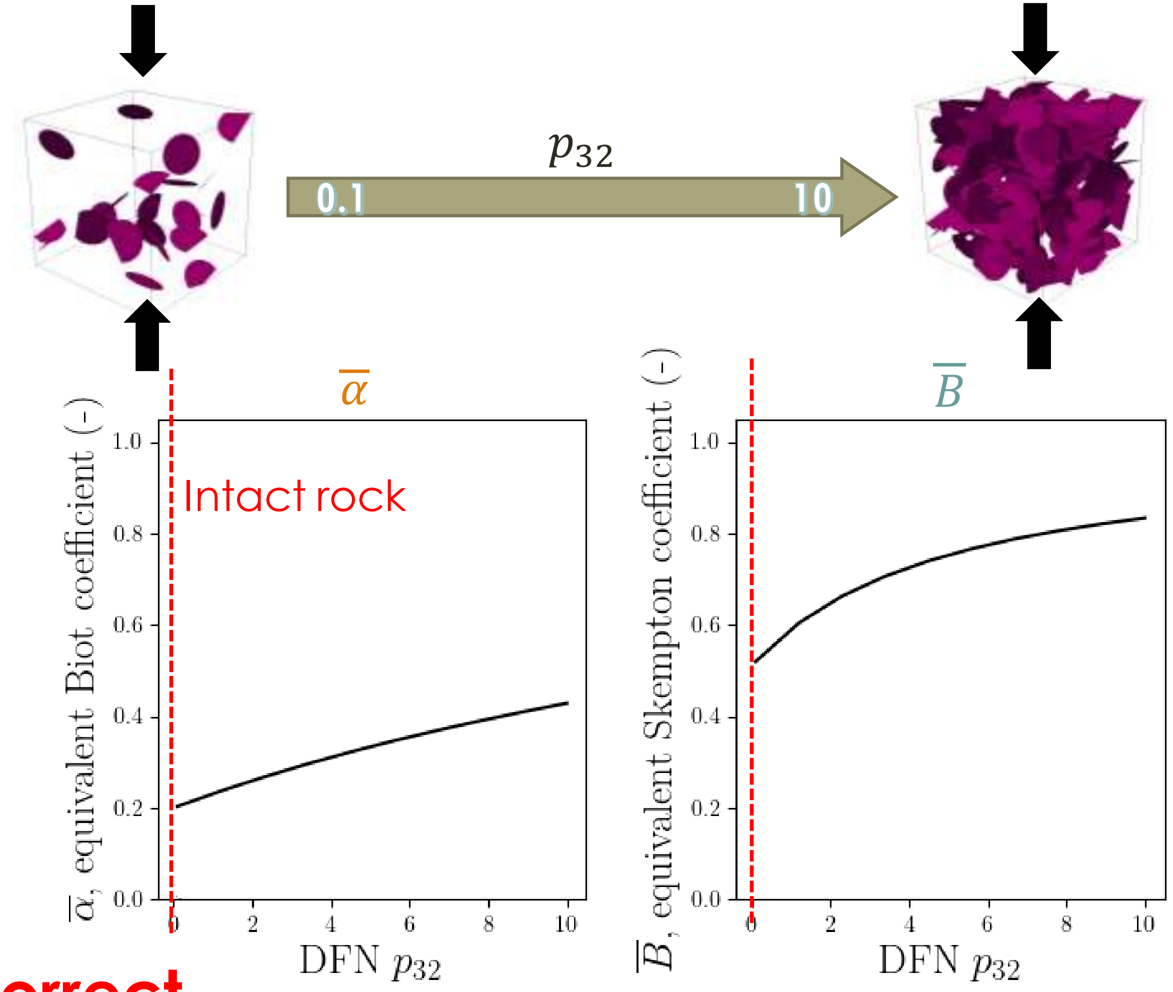
$\overline{\alpha}$ and $\overline{B}$ increase with fracture density because fractures increase the overall rock mass deformability

↓

smaller effective stresses $\Delta\sigma' \propto \left(1 - \overline{\alpha}\, \overline{B}\right)$

↓

smaller deformations?

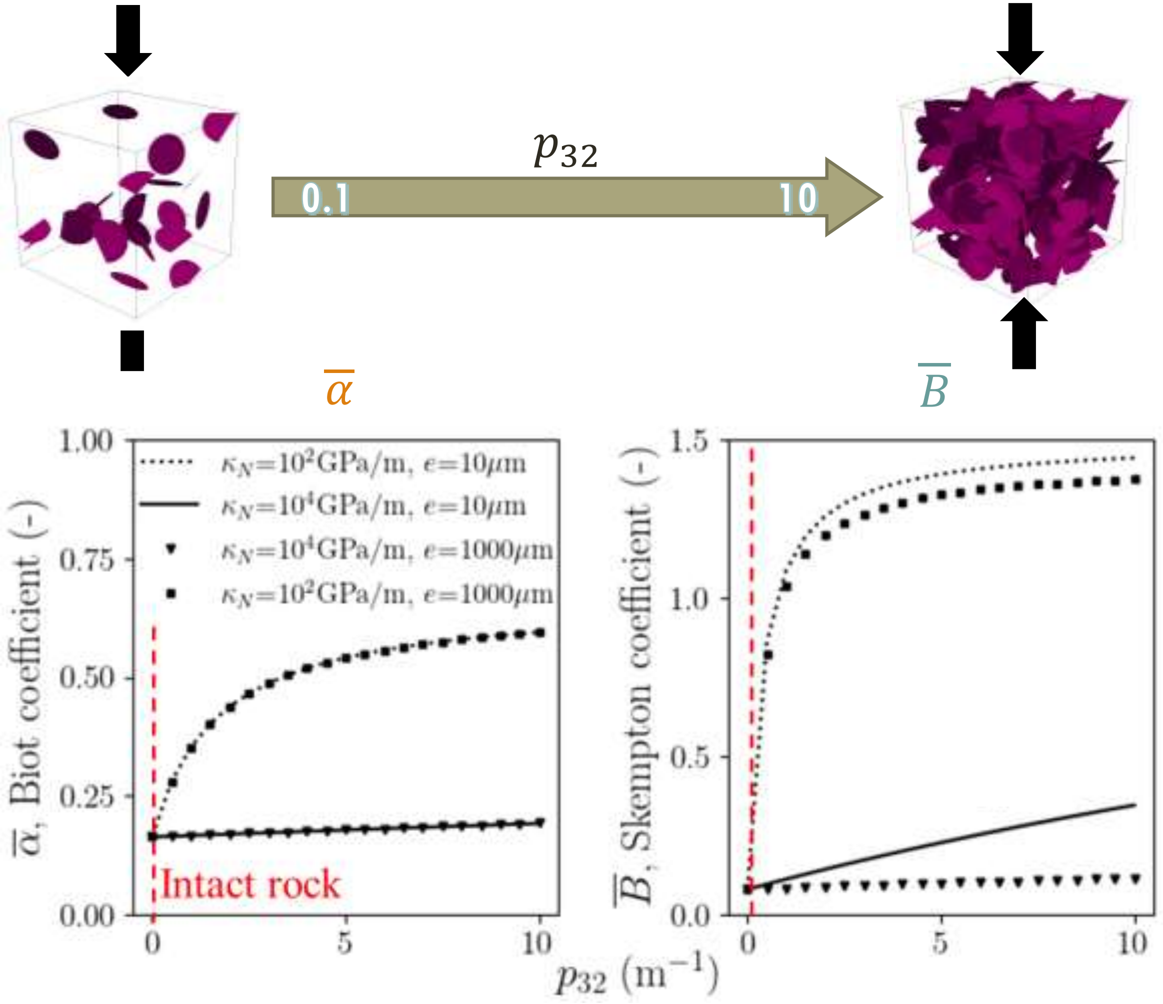


Neglecting fractures leads to incorrect estimations of the HM behavior

Variability of equivalent coefficients when fracture aperture (e) and stiffness (κ_N) are constant (ii)

$\bar{\alpha}$ and $\bar{B}$ depend on fracture stiffness and aperture

- They both decrease with increasing fracture stiffness
- Limited effects of aperture – $\bar{B}$ decreases with increasing fracture aperture



(De Simone *et al.*, RMRE 2023)

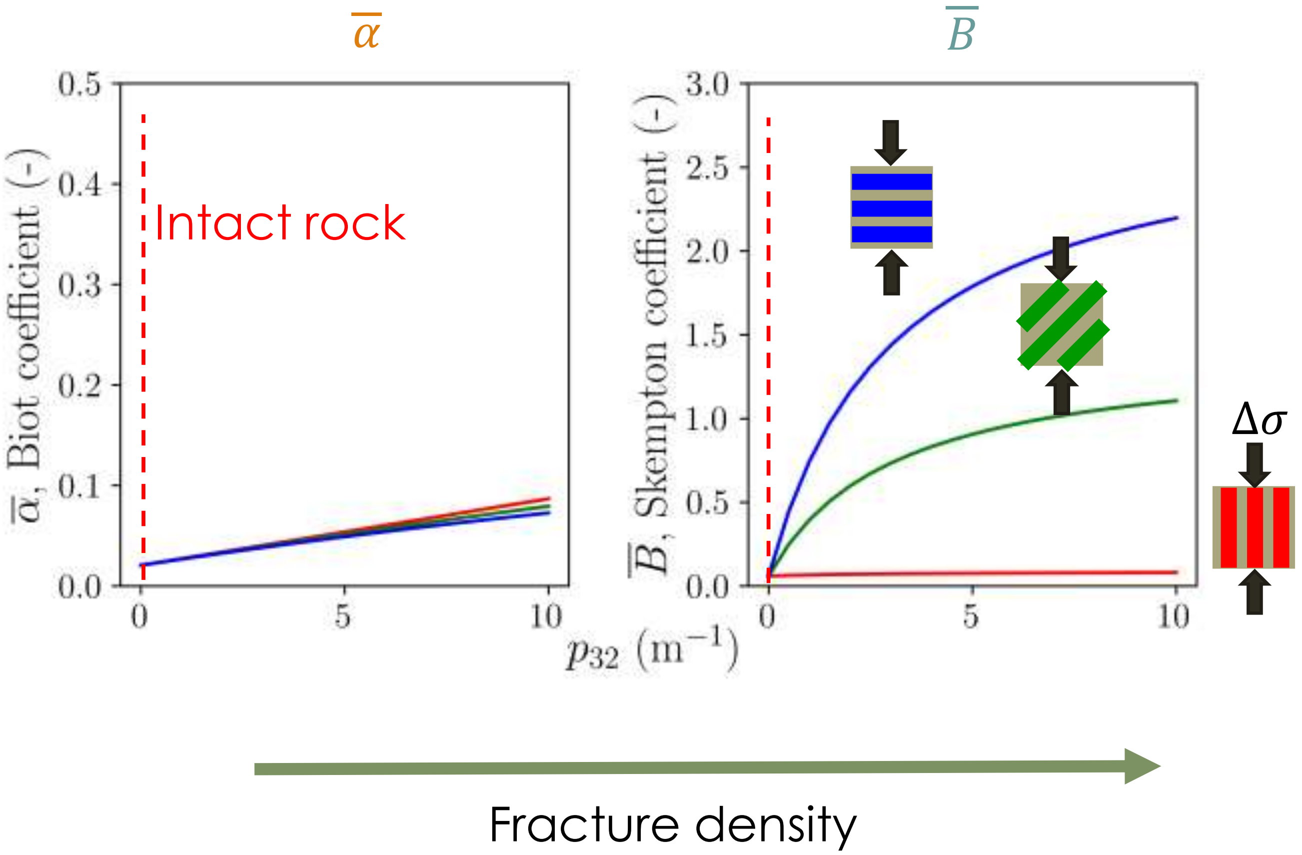
Uncertainty in characterizing the HM behavior

Variability of equivalent coefficients when fracture aperture (e) and stiffness (κ_N) are constant (iii)

$\overline{\alpha}$ and $\overline{B}$ depend on fracture orientation with respect to applied stress because orientation affects fracture contribution to the overall deforming capacity

$\overline{\boldsymbol{B}}$ is larger when the applied stress acts normal to the fractures

The opposite for $\overline{\boldsymbol{\alpha}}$ but very limited

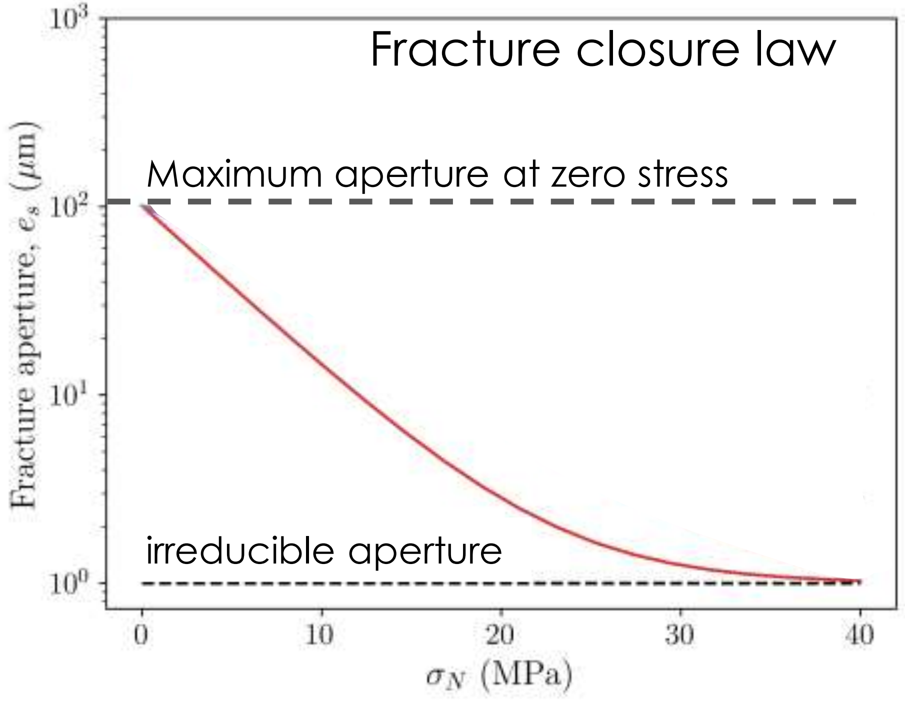


HM behavior is strongly anisotropic

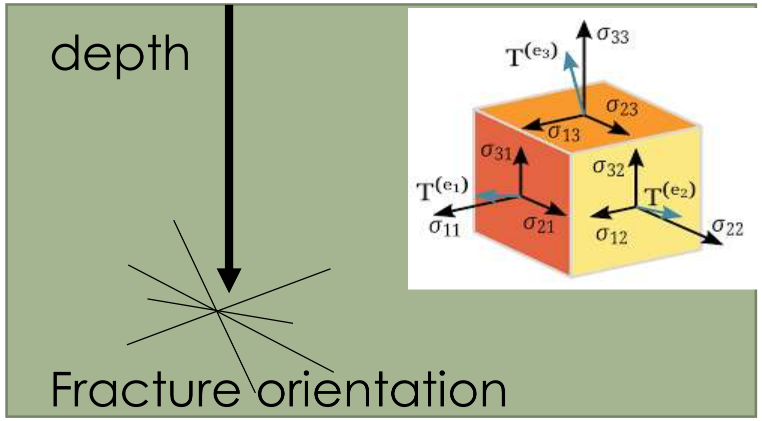
But fracture aperture (e) and stiffness (κ_N) are stress-dependent

Fracture aperture decreases **non-linearly** with increasing normal stress, because fracture stiffness increases as the normal stress increases

- hyperbolic (Goodman, 1976; Bandis et al., 1983)
- logarithmic (Evans et al., 1992)
- exponential law (Liu et al., 2012)



Fracture aperture and stiffness depend on:



In-situ stress tensor

$\Delta\sigma_N$ ↑ aperture ↓ stiffness ↑

Particularly affects the coefficients (they decrease with increasing stiffness)

Fracture orientation plays two effects

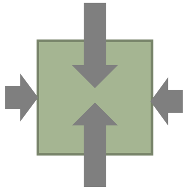
CONSTANT APERTURE AND STIFFNESS

- $\overline{\boldsymbol{\alpha}}$ and $\overline{B}$ depend on fracture **orientation** with respect to **applied stress** $\Delta\sigma$

STRESS-DEPENDENT APERTURE AND STIFFNESS

- $\overline{\boldsymbol{\alpha}}$ and $\overline{B}$ depend on fracture **orientation** with respect to **initial stress**

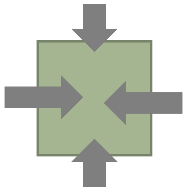
Initial stress



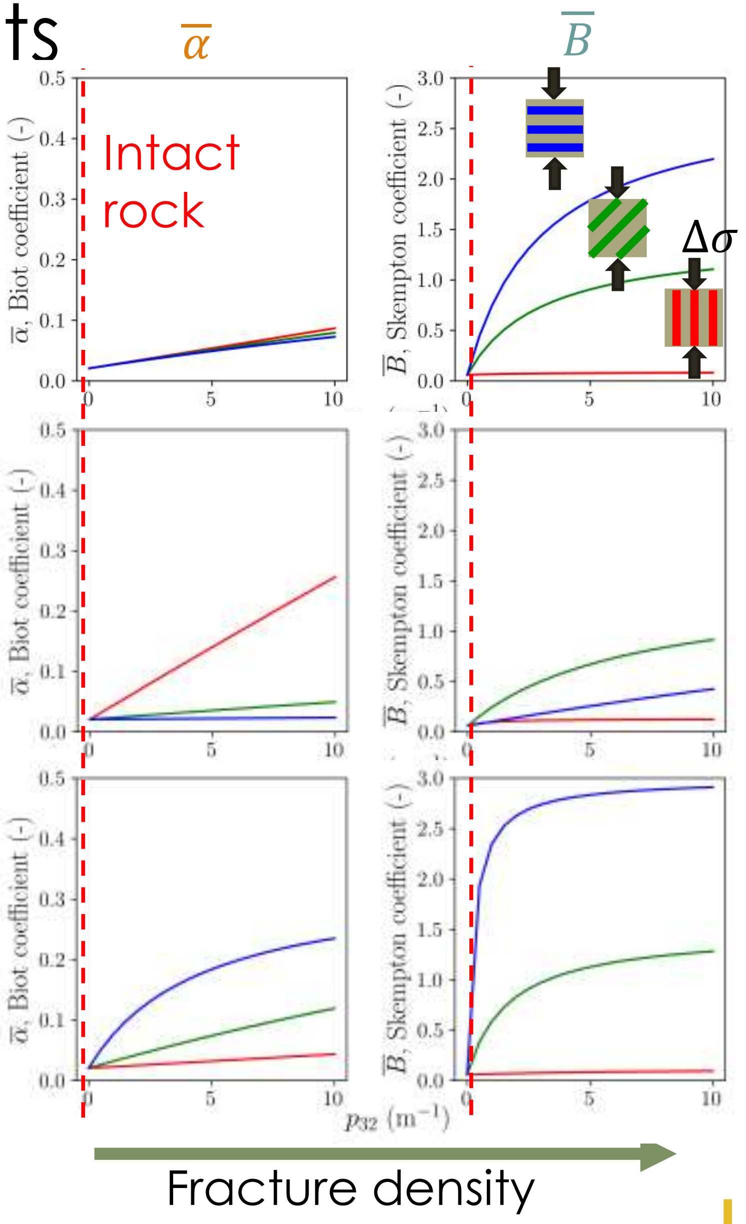
$\overline{\alpha}$: The effect of the orientation on $\Delta\sigma_N$ is **amplified** by its effect on κ_N

$\overline{B}$: The effect of the orientation on $\Delta\sigma_N$ is **counterbalanced** by its effect on κ_N

Initial stress

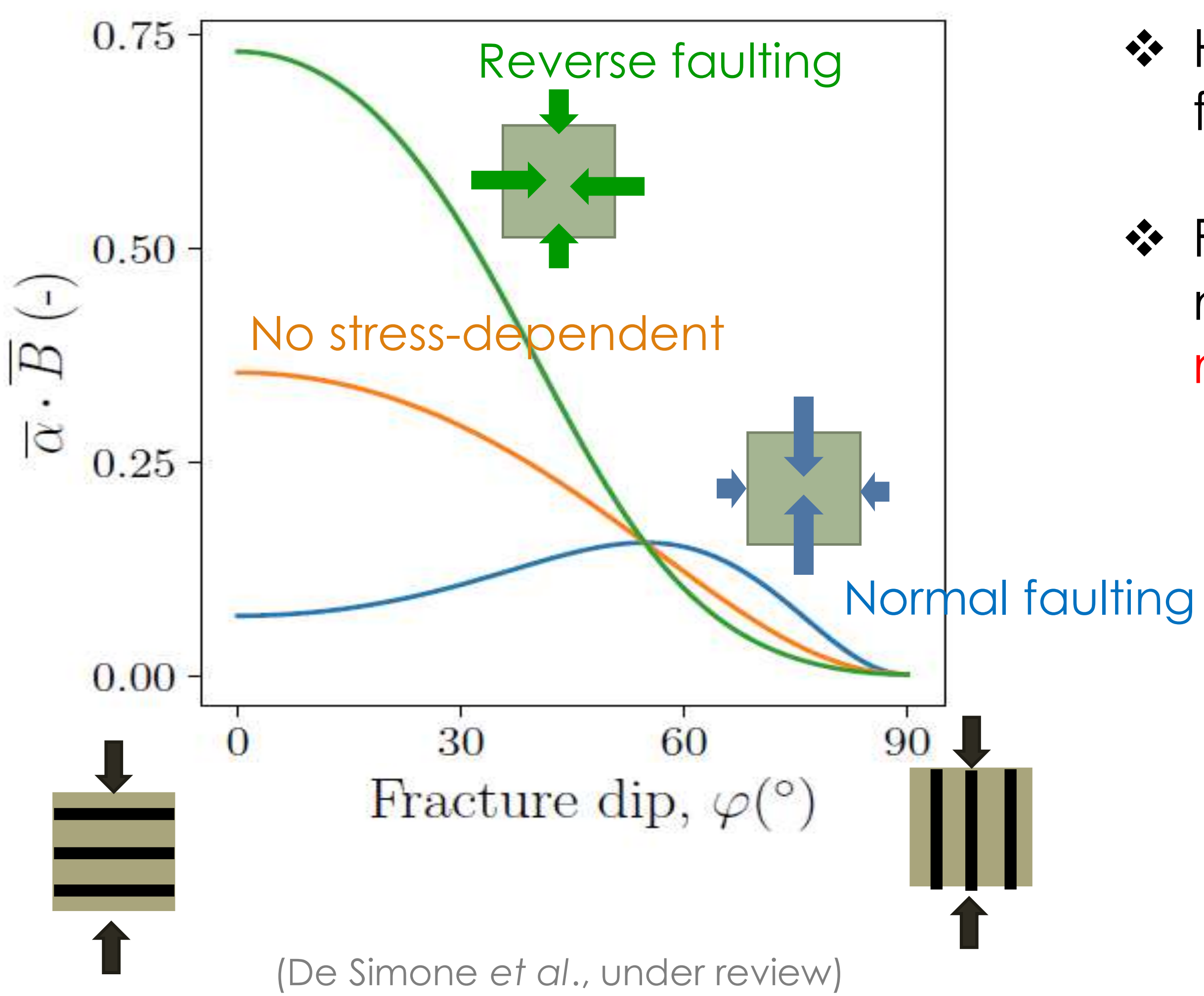


The opposite



HM behavior is even more anisotropic

In undrained conditions, the effective stress is minimized for:

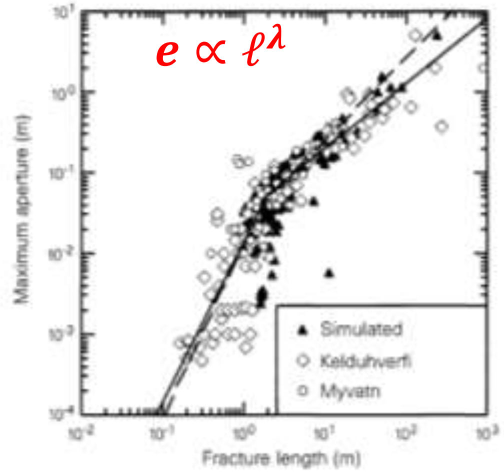


(De Simone *et al.*, under review)

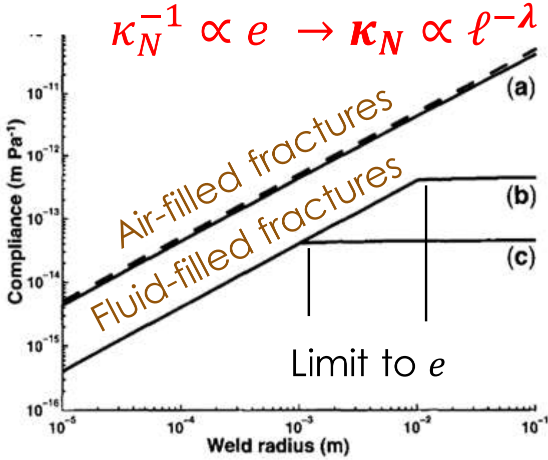
- ❖ Horizontal fractures under reverse faulting
- ❖ Fractures with dip=60° under normal faulting (which are the most critical for failure!)

Fracture aperture (e) and stiffness (κ_N) are size-dependent

Large fractures are more compliant and open

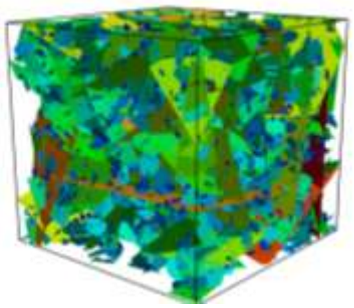


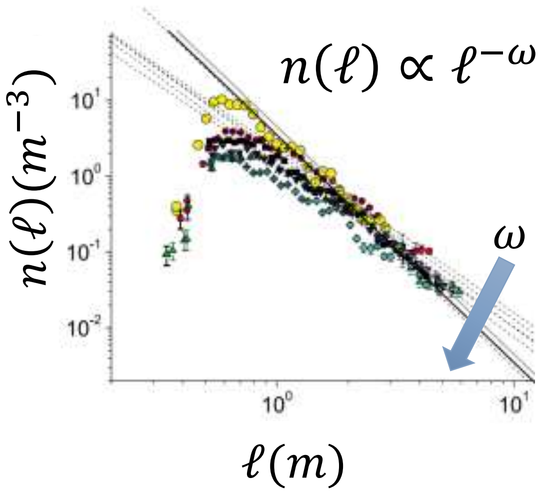
(Renshaw & Park, *Nature*, 1997)



(Worthington, *GSL*, 2007)

The distribution of fracture size follows a power law





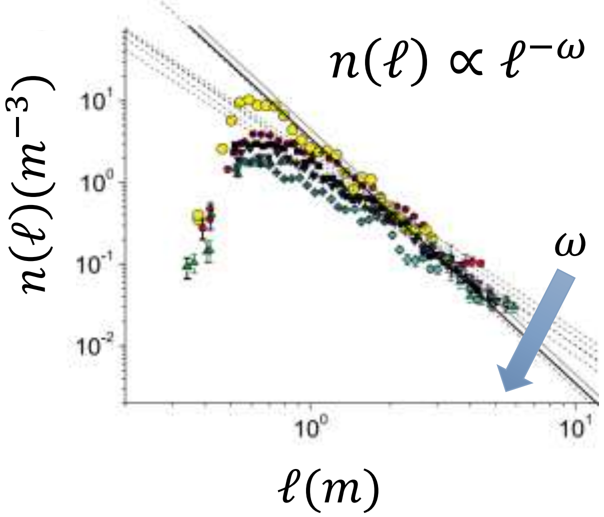
$$\sum_f \gamma^f = \sum_f \frac{S^f}{\kappa_N^f} \propto \int \ell^{2+\lambda}\, n(\ell)\, d\ell$$

Size distribution impacts the coefficients

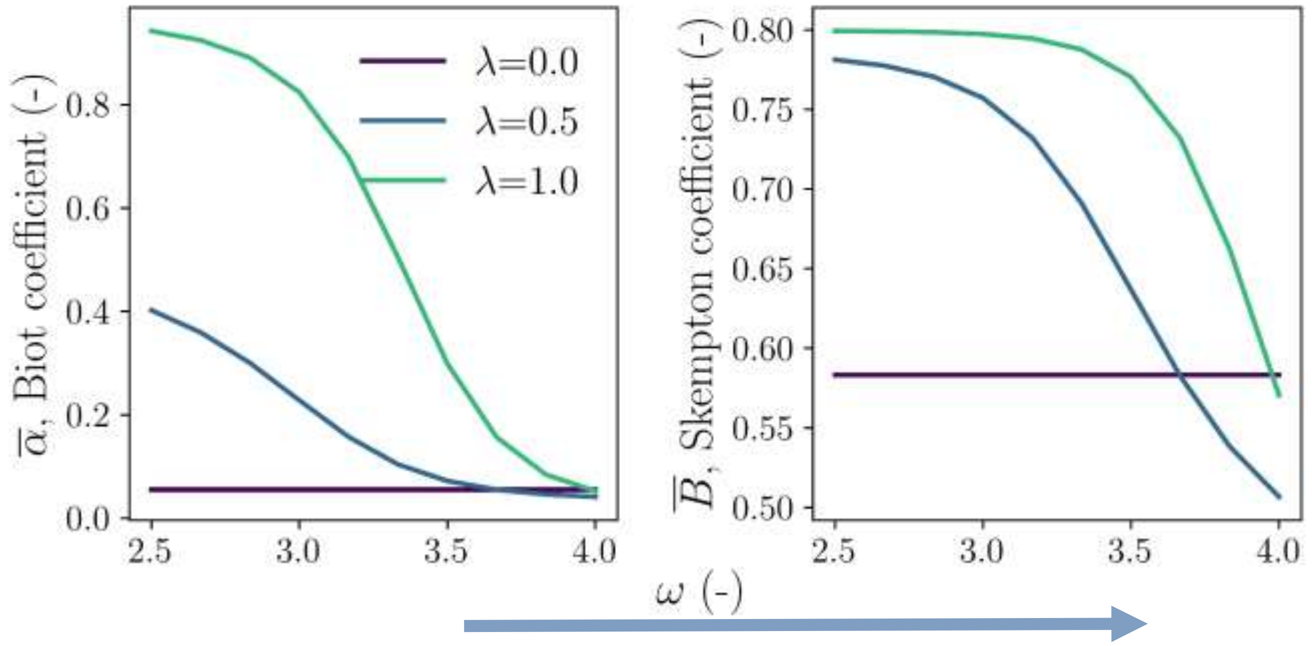
Assuming all fractures have the same orientation

$\boldsymbol{\kappa_N} \propto \ell^{-\lambda}$

$\boldsymbol{e} \propto \ell^{\lambda}$



$\overline{\alpha}$ and $\overline{B}$ larger in systems populated by a few large fractures than in systems with many small fractures (for equivalent p_{32})

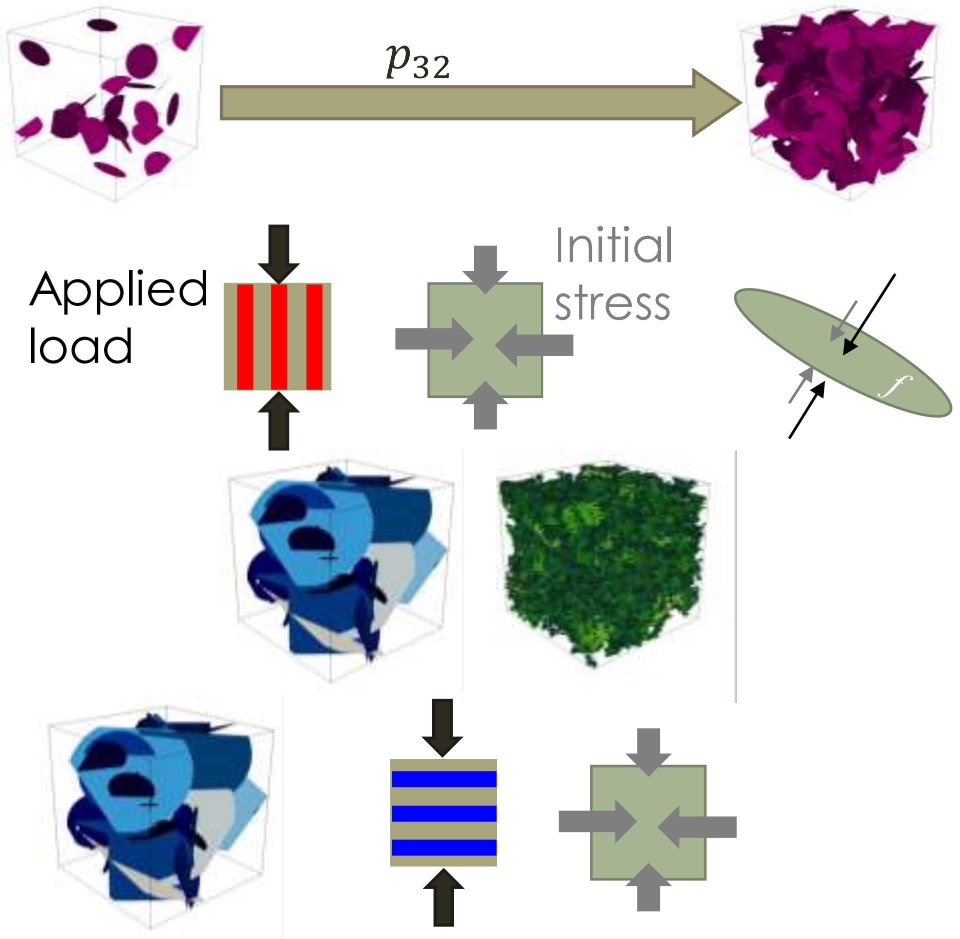


(De Simone *et al.*, under review)

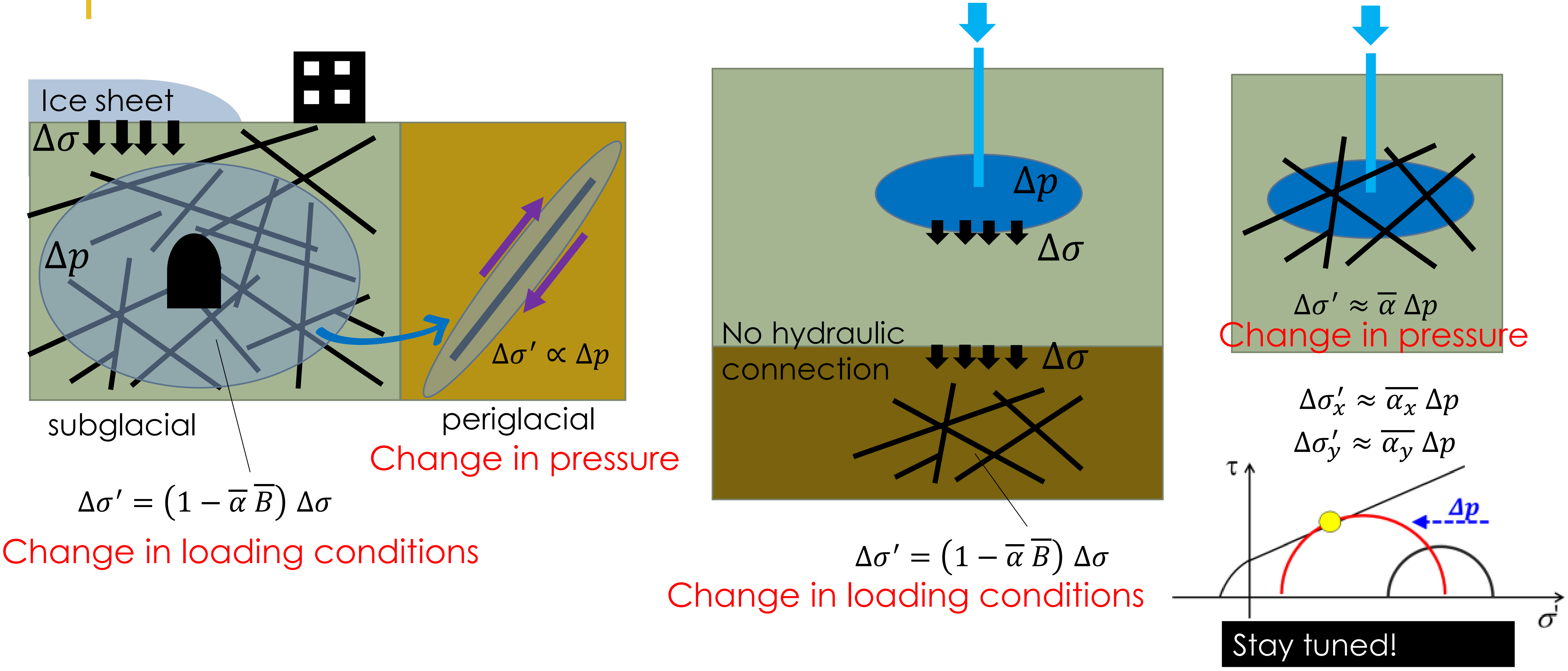
HM behavior determined by fracture size organization

Take-home messages

- Disregarding fractures lead to incorrect predictions: both coefficients increase with fracture **density**
- Strong **anisotropy**: Biot and Skempton coefficients depend on the **orientation** of the applied load, the initial stress and the fracture orientations
- Impact of the distribution of **fracture size**: Biot and Skempton coefficients are larger in systems populated by a few large fractures
- The fracture contribution is larger in systems containing **large fractures oriented parallel to the largest principal initial stress and normal to the applied stress**
- **Uncertainties** arise from parameters that are not yet constrained by measurements



Implications for HM behavior and induced seismicity



Thank you for your attention!

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